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13. ABSTRACT

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A PHASED ARRAY ANTENNA
WITH HEMISPHERIC SCAN
FOR SATELLITE TRACKING

THESIS

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A PHASED ARRAY ANTENNA WITH HEMISPHERIC SCAN FOR SATELLITE TRACKING

THESIS

Presented to the Faculty of the School of Engineering
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Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Thomas B. Markham
Captain USAF
Graduate Electrical Engineering

Preface

This thesis is the result of my study of high gain directive array antennas and my prior experience as a Satellite Operations Controller with the Air Force Satellite Control Facility. The phased array seemed to be the answer to the mechanical difficulties inherent to large highly directive parabolic antennas. However, a planar array cannot be scanned to the limits required unless some means of changing its position is provided. This does not eliminate the mechanical difficulties, only alters them. Therefore, I turned my attention to a non-planar array with as much symmetry as possible to reduce the complexity of the solution. With the help and guidance of Dr. Russell W. Taylor, to whom I am greatly indebted for his encouragement and understanding, I proceeded to formulate the solution of the far field pattern of an array with circular horizontal and elliptical vertical cross sections in a format suitable for solution by a high speed digital computer. The results of this study are presented in this paper. I would also like to thank Major John Pierce of the Air Force Institute of Technology Computer Center for his assistance in eliminating the many errors encountered in programming the solution and his patience during the many programs submitted for execution. I would also like to thank Mrs. Gloria McNally for her invaluable assistance in preparing this paper for submission.

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Abstract

The possibility of replacing large highly directive parabolic antennas for satellite tracking with a phased array is investigated. It is shown that a non-planar array with a circular horizontal cross section and an elliptical vertical cross section has a far field radiation pattern suitable for satellite tracking. The problem is formulated for solution by a high speed digital computer and an analysis of the performance of the optimal array design is presented.

A PHASED ARRAY ANTENNA WITH HEMISPHERIC SCAN FOR SATELLITE TRACKING

I. Introduction

Background

The Air Force Satellite Control Facility, (SCF), is charged with the responsibility of tracking, commanding, and gathering telemetry data from Department of Defense satellites. To accomplish this task the SCF is presently using a parabolic antenna with a diameter of sixty feet. A serious problem arises when tracking a low orbit (approximately 100 nautical miles) satellite during a near overhead pass. This problem is the extreme difficulty of rotating the large mass of the antenna as much as 180° in a matter of seconds without excessive lag or overshoot. This problem could be overcome by utilizing a fixed antenna with a steerable beam, i.e., a phased array. However, this solution poses another, equally serious, problem in that phased array technology at the present time does not include an array with acceptable beam characteristics at scan angles greater than approximately 60° from broadside. In order to accomplish the task of gathering data from a satellite it is necessary to maintain radio contact with it for as long as possible. In other words, the ground based tracking antenna must be capable of scan angles symmetric about the vertical axis for the entire range of 90° from the axis

Purpose

The purpose of this study is to investigate the feasibility of using a phased array to accomplish the SCF mission.

Approach

Since the technology to date does not include the large scan angles required, it was decided to abandon the planar phased array and investigate the

possibility of using a non-planar array. Since the requirement exists to scan 360° in azimuth, consideration was limited to geometry symmetric about the vertical axis. This indicated an array with a circular cross-section in the horizontal plane. The possible vertical cross-sections included conic sections, rectangles, trapezoids and truncated conic sections. Since consideration of all possible cross-sections was beyond the scope of this study, the geometry was further restricted to the most general conic section, the ellipse. With the general geometry of the array fixed, the CDC 6600 digital computer was programmed to calculate the far field radiation pattern of the specified array with the eccentricity of the ellipse, the number of elements in the array, and the phasing of the elements as input variables. The field pattern was calculated for various array designs at various scan angles in order to arrive at an optimal design.

II. Theory

Linear Array Theory

If an array is considered as a single source made up of differential radiators, then, ignoring the effects of mutual coupling, the resultant radiation pattern is a superposition of the field contributions of each elementary source. This means that, if the phase and amplitude of each element can be chosen such that the cancellation of the fields in all but the desired direction results, a highly directive antenna can be constructed. It is found that, in general, the desired results can be achieved by using identical elements equally spaced. By using this configuration, the total array pattern can be obtained from the product of the radiation pattern of the individual element and the pattern of an identical array of isotropic radiators, as will be shown in the discussion to follow (Ref 7).

Consider the simplest array of all, an array of two identical infinitesimal current elements located on the z-axis with a spacing, d, as shown in Fig. 1. The field at the point, P, far removed from the array, produced by the element located at the origin is expressed by the equation

$$E_\theta = \frac{i\, k_o Z_o I_o \, \Delta Z}{4\pi r} \sin\theta \, e^{-i k_o r} \tag{1}$$

where k_o is the free space wave number, Z_o is the characteristic impedance of free space, I_o is the excitation current of the element and ΔZ is the length of the element. The field at P due to the other element can be expressed by the same equation with r replaced by R. From the law of cosines

$$R = (r^2 + d^2 - 2\, r\, d \cos\theta)^{\frac{1}{2}} \tag{2}$$

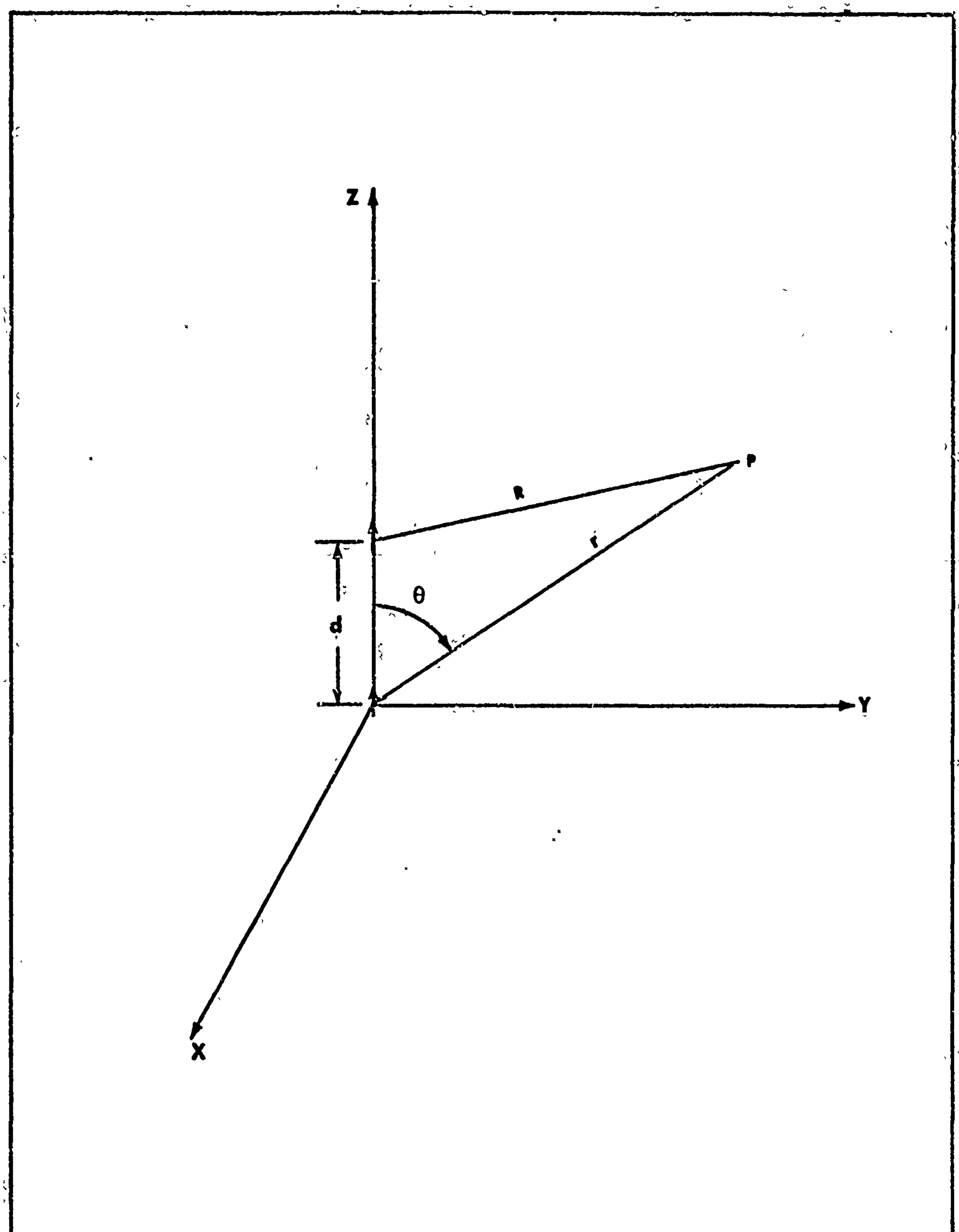

Fig. 1. Array of Two Elementary Current Elements

or, alternatively,

$$R = r\left[1 - \frac{2\,d\cos\theta}{r} + \left(\frac{d}{r}\right)^2\right]^{\frac{1}{2}} . \tag{3}$$

This latter equation may be expanded by means of the binomial theorem in powers of d/r. Since it is specified that $r \gg d$, the first two terms of the expansion are sufficient for phase calculations with negligible error. In addition, $R \approx r$ for far field calculations. Using these approximations the field at P due to element number 2 is expressed by

$$E_\theta = \frac{j\,k_o\,Z_o\,I_o\,\Delta Z}{4\pi r}\,\sin\theta\, e^{-j\,k_o\,(r - d\cos\theta)} . \tag{4}$$

By superposition the total field at P is given by

$$E_\theta = \frac{j\,k_o\,Z_o\,I_o\,\Delta Z}{4\pi r}\left[e^{-j\,k_o\,r} + e^{-j\,k_o\,(r - d\cos\theta)}\right]\sin\theta$$

or

$$E_\theta = \frac{j\,k_o\,Z_o\,I_o\,\Delta Z}{4\pi r}\, e^{-j\,k_o\,r}\left[1 + e^{j\,k_o\,d\cos\theta}\right]\sin\theta . \tag{5}$$

Figure 2 shows graphically the variation of E_θ with θ.

It is easily seen from the above analysis that if arbitrary identical antenna elements replace the infinitesimal current elements above, the resultant field could be written

$$E_\theta = f(\theta, \varphi)\frac{e^{-j\,k_o\,r}}{r}\left[1 + e^{j\,k_o\,d\cos\theta}\right] . \tag{6}$$

where $f(\theta, \varphi)\exp(-j\,k_o\,r)/r$ represents the field due to one element located at the origin of the coordinate system.

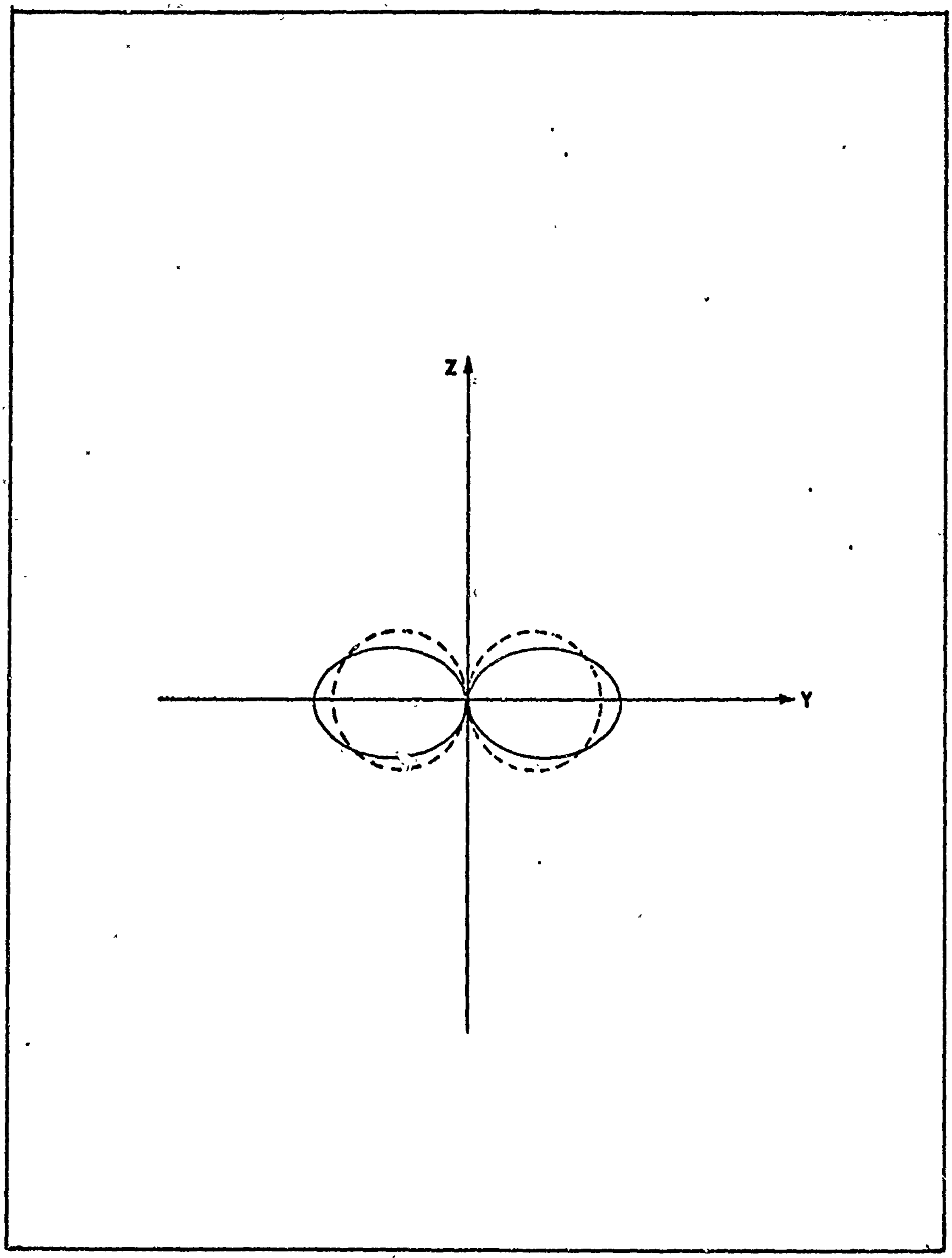

Fig. 2. Radiation Pattern of Single Dipole (----) and Array of Figure 1 (——)

Generalizing these results still further, for an array of N identical antenna elements spaced a uniform distance, d, apart along the z-axis, the far field can be expressed as

$$E_\theta = f(\theta,\varphi)\frac{e^{-jk_o r}}{r}\left[1+\sum_{n=1}^{N-1} e^{jk_o n d\cos\theta}\right] \tag{7}$$

with the restriction that $r \gg Nd$ for the far field approximations to hold. This expression for the far field may be considered as the product of the element factor, $f(\theta,\varphi)\exp(-jk_o r)/r$, and an array factor, A, with

$$A = 1 + \sum_{n=1}^{N-1} e^{jk_o n d\cos\theta} .$$

If the amplitude and phase of the individual elements are varied such that the n-th element has amplitude, C_n, and phase, $\exp(j\alpha_n)$, referenced to the element at the origin, the array factor, A, becomes

$$A = 1 + \sum_{n=1}^{N-1} C_n e^{jk_o n d\cos\theta}\, e^{j\alpha_n} . \tag{8}$$

Now consider the linear array of Fig. 3 with a uniform phase progression denoted by

$$\alpha_n = -n\, d\, k_o \cos\theta_o \tag{9}$$

where θ_o is a constant denoting the desired direction of the main beam. Set $C_n = 1$ for all n. Then the magnitude of A is given by

$$|A| = \left| 1 + e^{jk_o d(\cos\theta - \cos\theta_o)} + e^{jk_o 2d(\cos\theta - \cos\theta_o)} + \dots + e^{jk_o (N-1) d(\cos\theta - \cos\theta_o)} \right| . \tag{10}$$

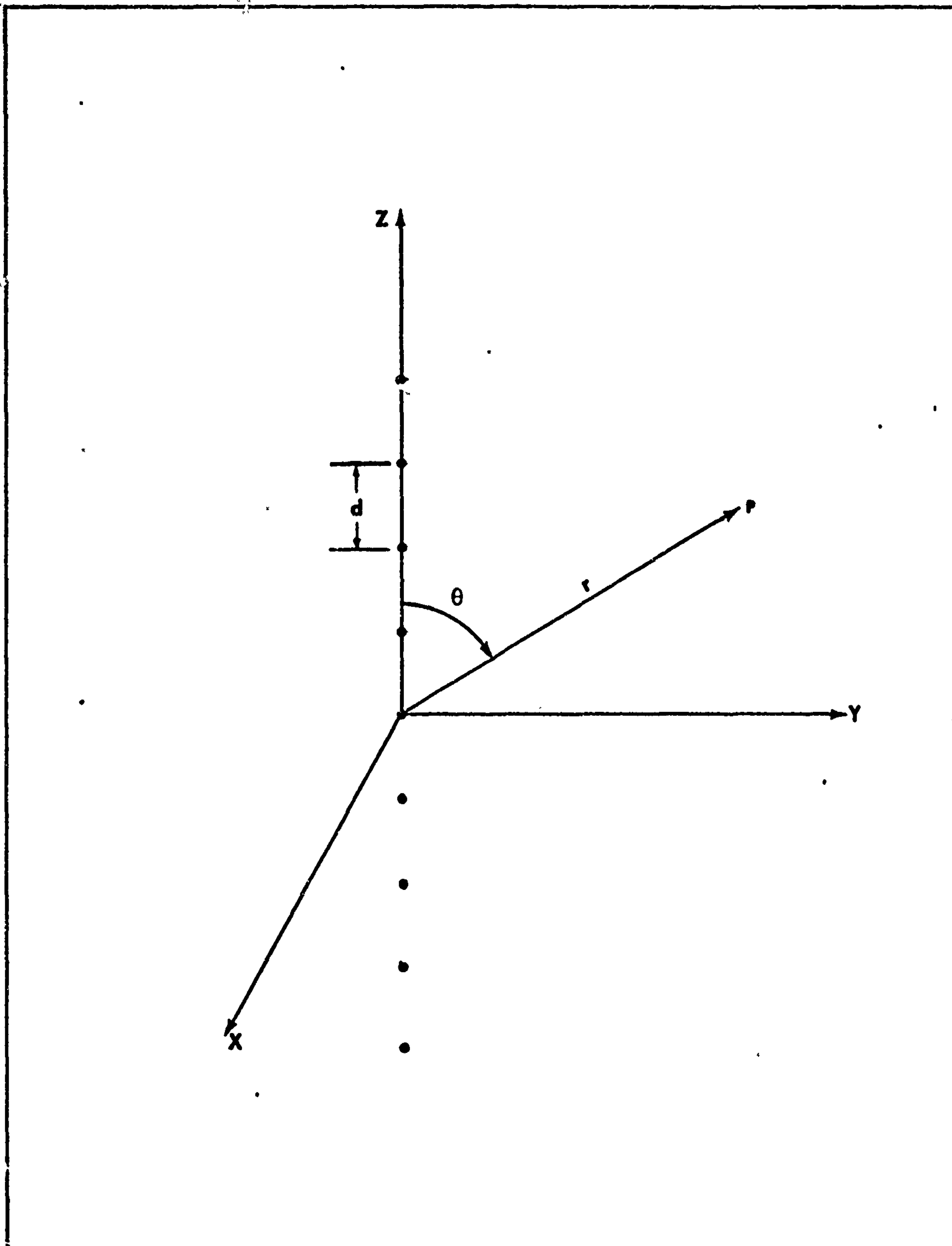

Fig. 3. Linear Array of Elementary Radiators

Equation (10) is a geometric progression with a ratio $\exp[j\, k_o\, r(\cos\theta - \cos\theta_o)]$ which can be expressed as

$$|A| = \left| \frac{\sin \dfrac{N k_o d (\cos\theta - \cos\theta_o)}{2}}{\sin \dfrac{k_o d (\cos\theta - \cos\theta_o)}{2}} \right| . \tag{11}$$

Let $x = k_o d (\cos\theta - \cos\theta_o)$. Then it is evident that as x approaches zero, A approaches N. Nulls in the pattern occur when $|A| = 0$, or when $Nx/2 = \pi, 2\pi, 3\pi, \ldots, (N-1)\pi$. Since the maximum value of $|A|$ occurs at $x = 0$, the main beam is indeed in the direction $\theta = \theta_o$. This fact indicates that the direction of the main beam may be shifted by altering the phase shift of the elements, α_n. It is now of interest to examine the field pattern as the direction of the main beam is shifted through large angles. (Refer to Fig. 4.) Designating the position of the first null to the left of the main beam as θ^+ and the first null to the right as θ^- it is seen that

$$k_o d [\cos(\theta_o + \theta^+) - \cos\theta_o] = \frac{2\pi}{N} \tag{12}$$

and

$$k_o d [\cos(\theta_o - \theta^-) - \cos\theta_o] = \frac{-2\pi}{N} . \tag{13}$$

Assuming θ^+ and θ^- to be very small (as indeed will be shown to be true for large N) the approximations $\cos\theta = (1 - \theta^2/2)$ and $\sin\theta = \theta$ may be used. With these substitutions, equations (12) and (13) become

$$k_o d [-\theta^+ \sin\theta_o - \frac{(\theta^+)^2}{2} \cos\theta_o] = \frac{2\pi}{N} \tag{14}$$

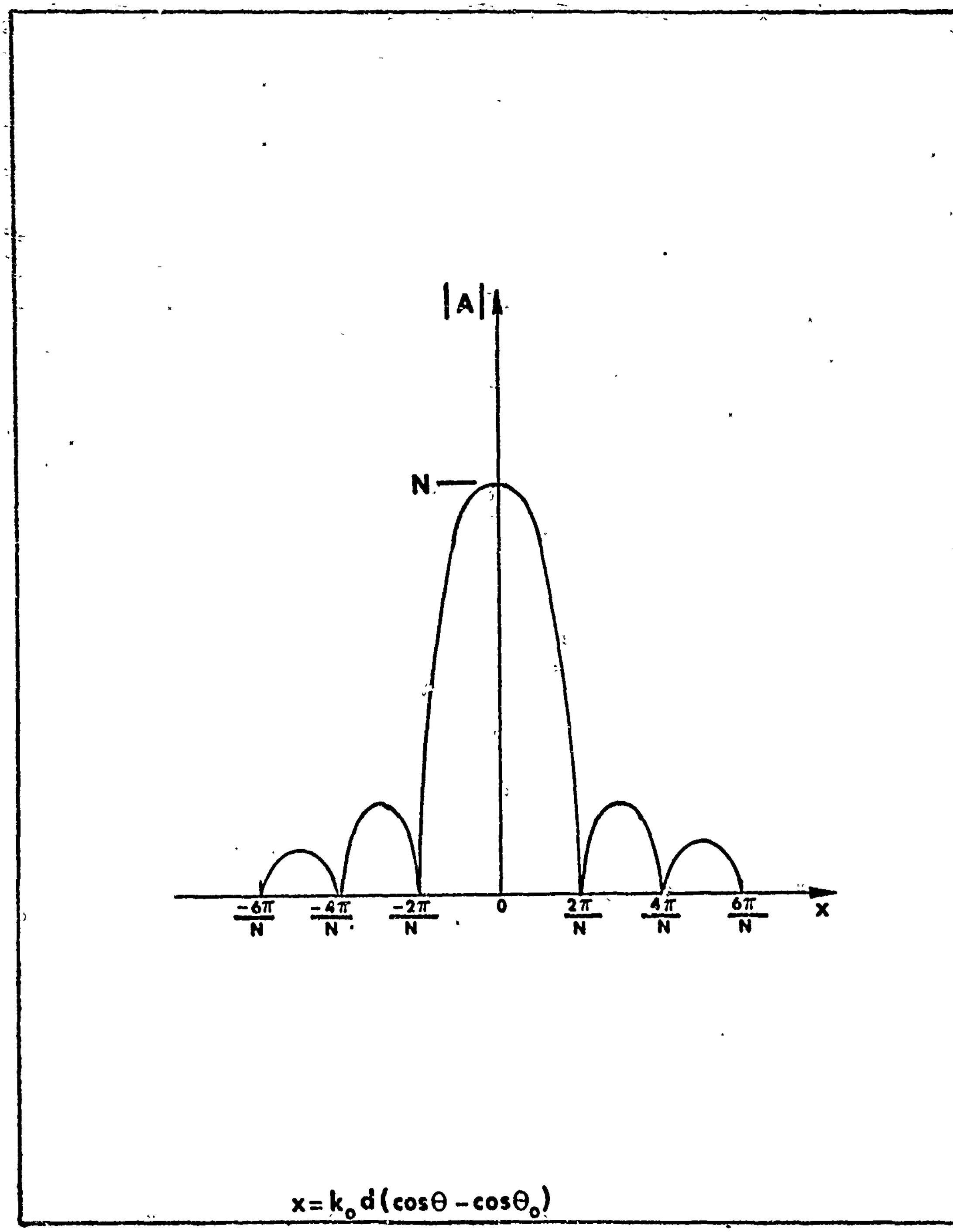

Fig. 4. Array Factor of Array in Fig. 3

and

$$k_o d\,[\,\theta^- \sin\theta_o - \frac{(\theta^-)^2}{2}\cos\theta_o\,] = \frac{-2\pi}{N}. \qquad (15)$$

Assuming $\theta_o >> \theta^{\pm}$, the quadratic terms in equations (14) and (15) can be dropped with the resulting expressions for θ^+ and θ^-.

$$\theta^+ = \theta^- = \frac{2\pi}{N k_o d \sin\theta_o} = \frac{\lambda_o}{N d \sin\theta_o}. \qquad (16)$$

Thus it is seen that if Nd/λ_o is large and θ_o sufficiently large, θ^+ and θ^- are indeed small as was assumed earlier. It is also evident that as θ_o varies from $\pi/2$, θ^+ and θ^- increase until the assumptions made to obtain equation (16) are no longer valid, i.e., the quadratic terms in equations (14) and (15) can no longer be dropped. For $\theta_o = 0$, it is seen that

$$\theta^+ = \theta^- = (2\lambda_o/Nd)^{\frac{1}{2}}. \qquad (17)$$

This condition results in an endfire array. While a linear array can produce a narrow beam in the θ-direction, Fig. 5 shows that the beam is in fact symmetric about the endfire axis. In addition, the broadening of the beam as θ_o approaches zero renders it unsuitable for large scan angles.

Planar Arrays

The above theory of linear arrays is readily expanded to the case of the two-dimensional array of equally spaced arrays of equally spaced elements (see Fig. 6). This array may be considered to be a linear array along the y-axis whose elements are linear arrays oriented in the z-direction. From the above analysis it is seen that the far field of this array is the product of an array factor, A_v, and an element factor, A_h, where the element factor is the product of the individual element factor and the array factor of a linear array is the z-direction.

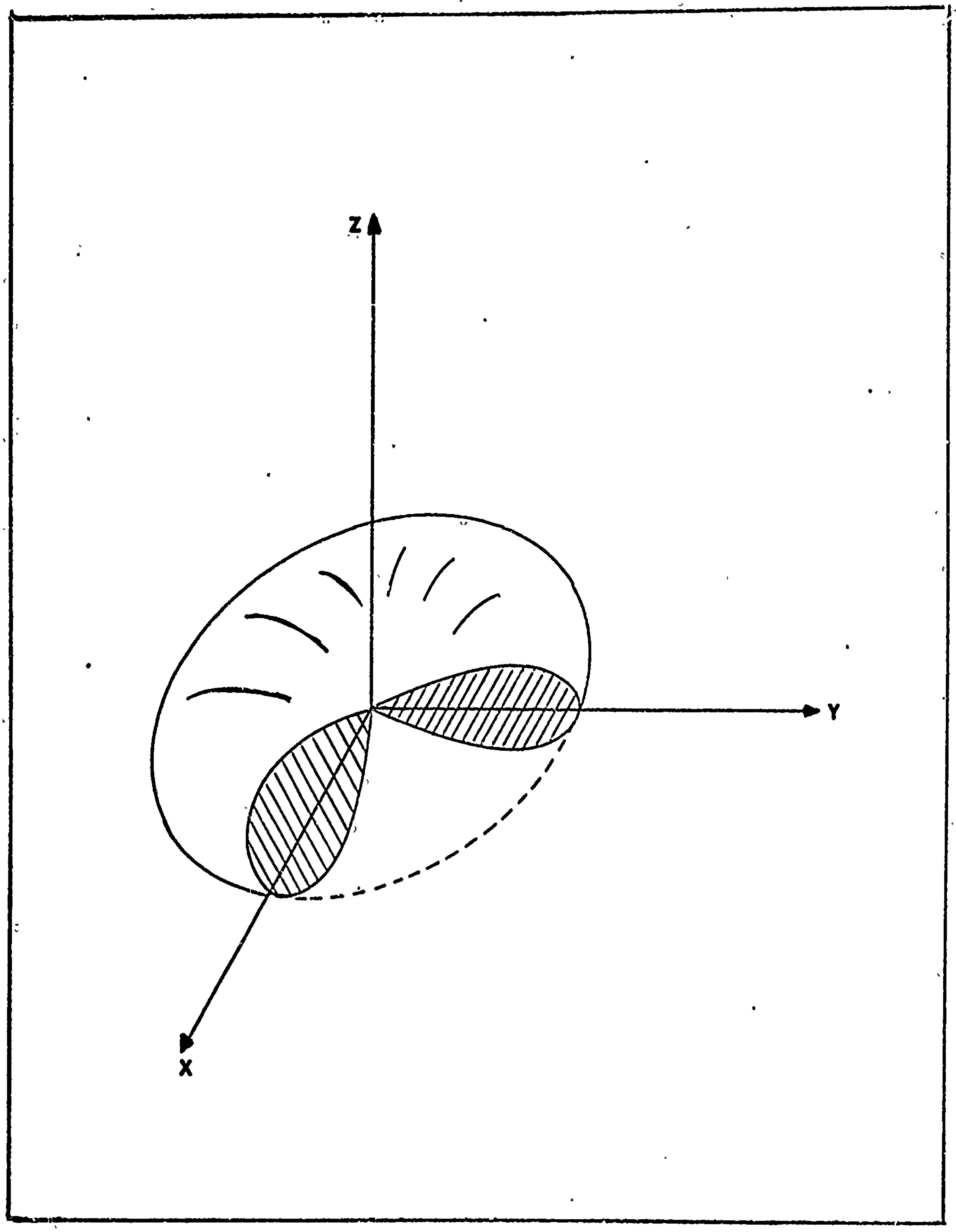

Fig. 5. Radiation Pattern of Linear Array of Fig. 3

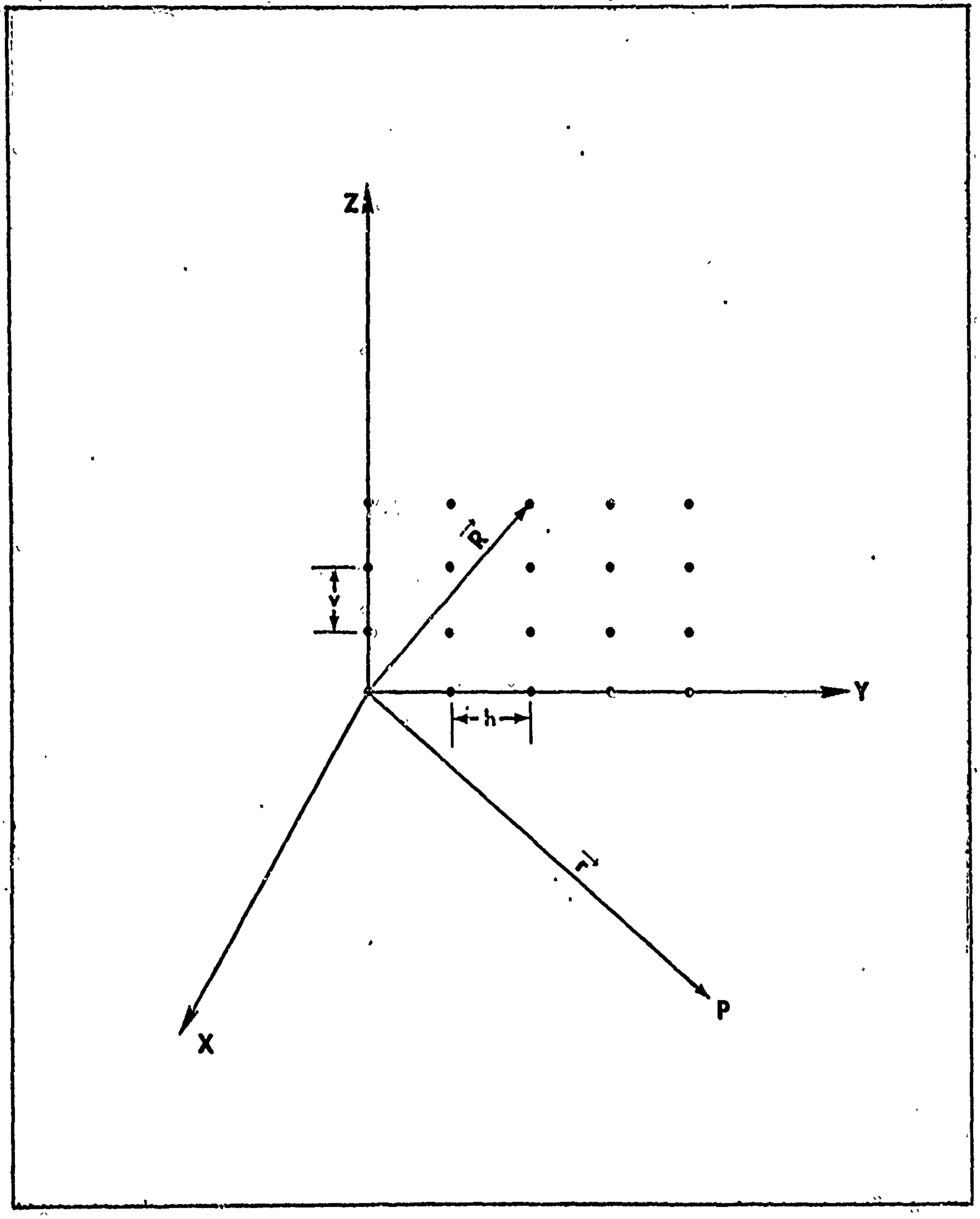

Fig. 6. Two Dimensional Linear Array

$$E_\theta = f(\theta,\varphi)\frac{e^{-\theta k_o r}}{r}\left[1+\sum_{m=1}^{M-1} e^{jm(k_o v\cos\theta-\beta)}\right]$$

$$\left[1+\sum_{n=1}^{N-1} e^{jn(k_o h\sin\theta\sin\varphi-\alpha)}\right] \tag{18}$$

where α and β are the progressive phase shifts in the horizontal and vertical directions respectively. Alternatively this can be written

$$E_\theta = f(\theta,\varphi)\frac{e^{-jk_o r}}{r}\sum_{m=0}^{M-1} e^{jm(k_o v\cos\theta-\beta)}\sum_{n=0}^{N-1} e^{jn(k_o h\sin\theta\sin\varphi-\alpha)}$$

$$= f(r,\theta,\varphi)\,A_h\,A_v \tag{19}$$

where A_v is the array factor calculated above in the analysis of the linear array of elements along the z-axis, and A_h is the array factor of an identical array oriented along the y-axis. Figure 7 shows that the resulting field from the two-dimensional array is no longer symmetric about the endfire axis (if in fact an endfire axis could be defined) but is a pencil beam on either side of the array.

<u>Volume Arrays</u>

Logic suggests that an extension of the theory of linear arrays to the case of a three-dimensional array made up of a linear array of two-dimensional arrays described above should yield a single pencil beam which could be scanned in any direction desired. Indeed, extending linear array theory to the volume array of Fig. 8 yields an expression for the far field

$$E_\theta = f(r,\theta,\varphi)\,A_x\,A_y\,A_z \tag{20}$$

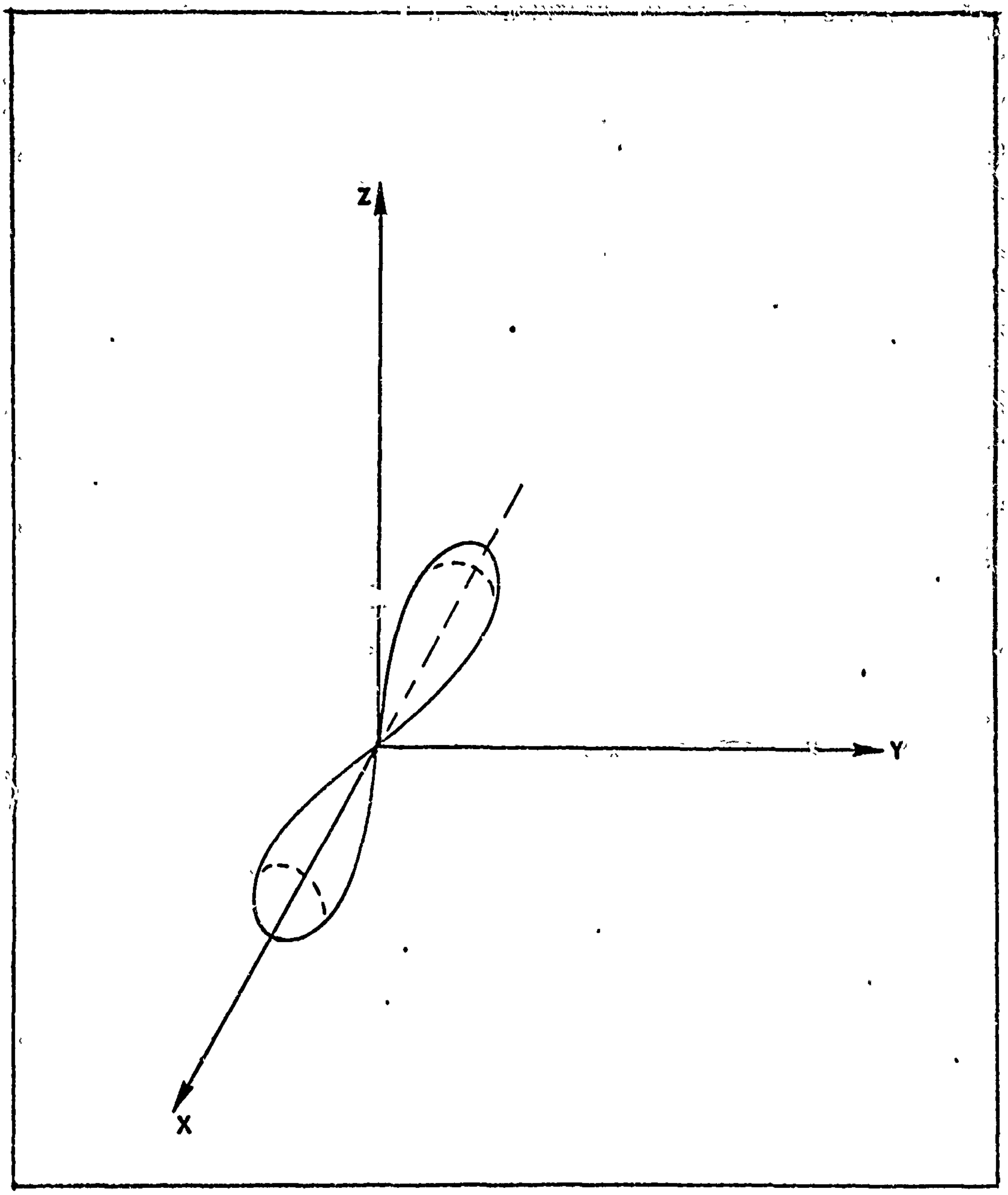

Fig. 7. Radiation Pattern of Array of Fig. 6

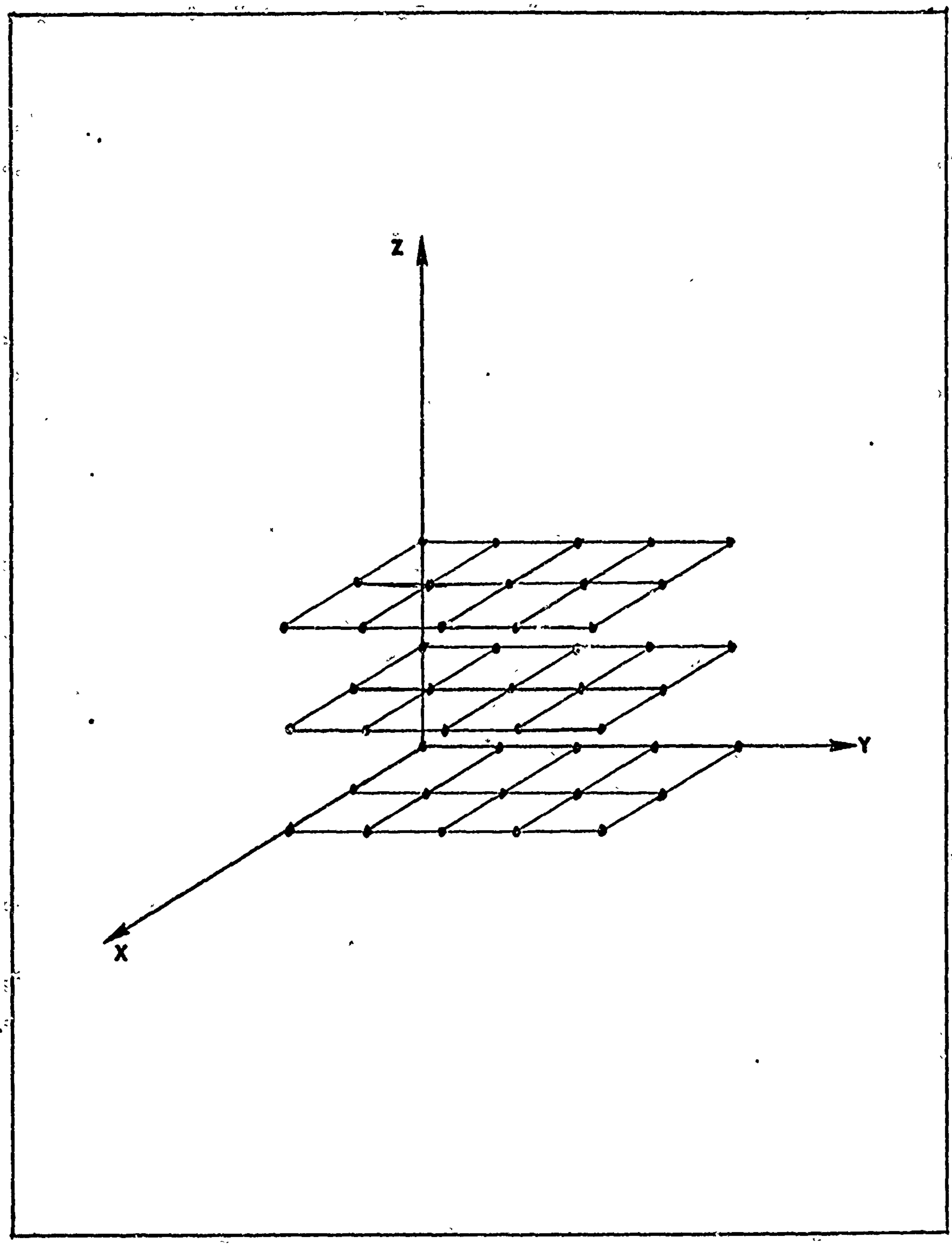

Fig. 8. Volume Array of Elementary Radiators

where

$$A_x = \sum_{m=0}^{M} e^{j\, m\,(k_o\, a \sin\theta \cos\varphi - \alpha)} \tag{21}$$

$$A_y = \sum_{n=0}^{N} e^{j\, n\,(k_o\, b \sin\theta \sin\varphi - \beta)} \tag{22}$$

and

$$A_z = \sum_{p=0}^{P} e^{j\, p\,(k_o\, c \cos\theta - \zeta)} \tag{23}$$

with α, β, and ζ the progressive phase shifts in the x, y, and z directions respecively. It is easily seen that an analytic solution to equation (20) would be extremely difficult for a large array. However, this equation lends itself readily to solution by a high speed digital computer.

Mutual Coupling

It has been shown that the radiation pattern and input impedance of individual elements in an array are changed by the effects of mutual coupling. It is also known that these effects become more pronounced at large scan angles (Ref 9). Cheng states:

> "To the author's knowledge, no work has been published on array synthesis or optimization for mutually coupled aperture-type radiators." (Ref l:1671)

For this reason effects of mutual coupling were not considered in this study.

III. A Non-planar Array for Satellite Tracking

Design Considerations

The task of tracking an orbiting satellite dictates certain restrictions on the geometry of the array to be used. Complete symmetry about the vertical axis specifies the optimum horizontal cross-section as a circle. The requirement to track from horizon to horizon precludes the use of a planar array due to the unsatisfactory beam characteristics at large scan angles. For these reasons the geometry was chosen to be a surface described by the equation

$$x^2 + y^2 + z^2/E = R^2 \tag{24}$$

It can be seen from the analysis of the volume array that the beamwidth is a function of the number of elements in the array, the spacing of the elements, the wavelength of the radiated wave, and the scan angles θ and φ. Due to the symmetry chosen, the φ-dependence of the beamwidth was eliminated. Also the operating frequency (1.7 to 2.3 GHz) fixed the wavelength.

The operating frequency also suggested the use of rectangular aperatures as the basic radiating element. Design consideration was limited to a rectangular aperture operating in the TE_{10} mode. This restriction dictated an aperture 0.8 wavelength wide and 0.4 wavelength high. This further indicated a spacing of the elements of one wavelength in the φ-direction and 0.5 wavelength in the θ-direction. The radius at the base of the antenna depends on the number of elements and the spacing. These restrictions fixed all variables with the exception of the eccentricity of the vertical cross-section and the number of elements at the base of the array. Figure 9 shows the basic geometry considered in this study.

Field Calculations

With the geometry of the antenna specified, the field of the antenna can be calculated from the equation for the volume array

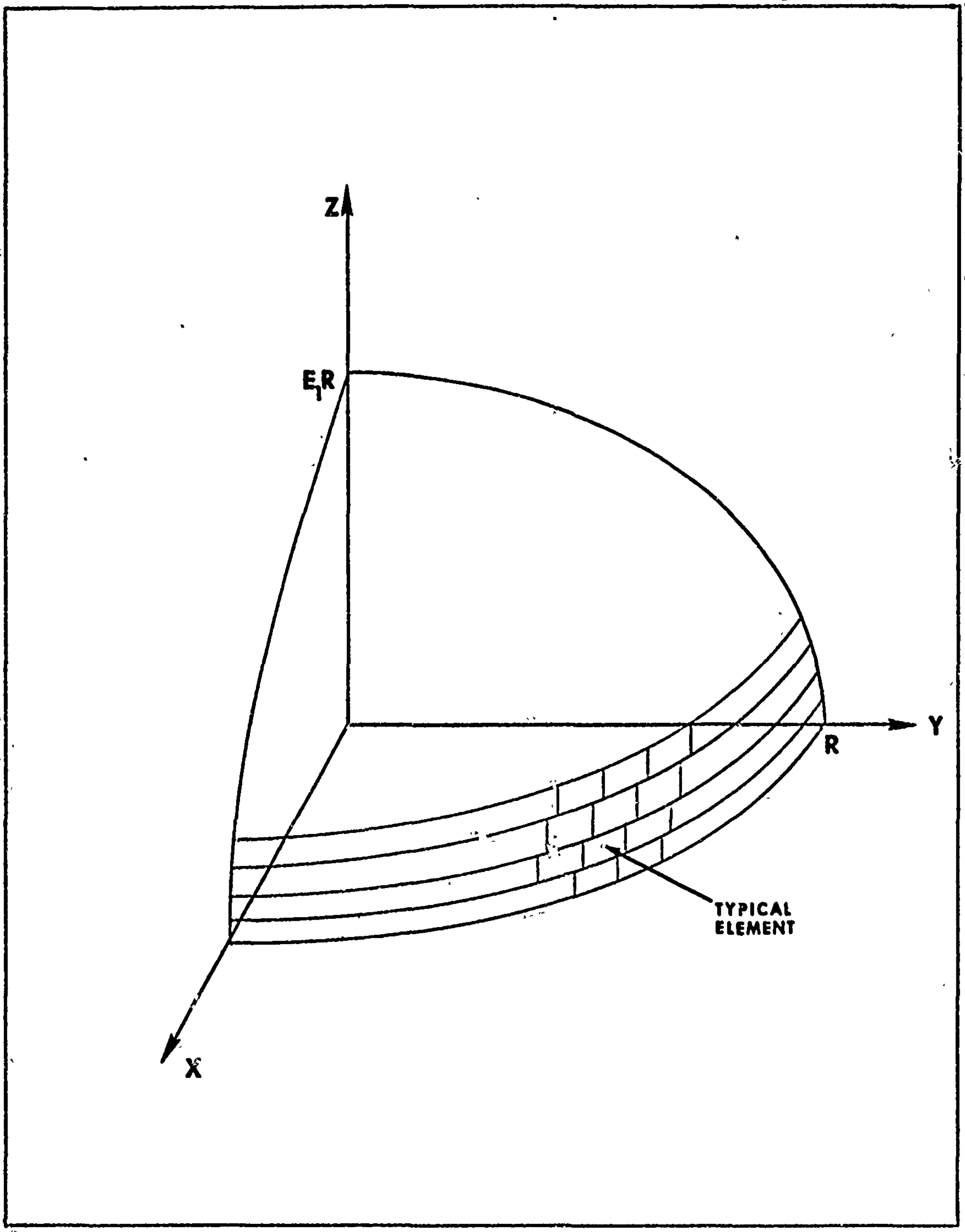

Fig. 9. Basic Geometry of Array Considered

$$E_\theta = \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{p=0}^{P} f_{mnp}(r,\theta,\varphi)\, C_{mnp}\, A_x A_y A_z \qquad (25)$$

where $f_{mnp}(r,\theta,\varphi)$ is a function of position of the mnp-th element in the array and $C_{mnp} = 0$ for all elements not on the surface described by equation (24). Equation (25) is now rewritten in polar form as

$$E_\theta = \frac{e^{-jk_o r}}{r} \sum_{\theta=0}^{\pi/2} \sum_{\varphi=0}^{2\pi} f(\theta,\varphi)\, e^{-jk_o f_1(\theta,\varphi)}\, e^{jk_o f_2(\theta,\varphi)} \qquad (26)$$

where f, f_1, and f_2 are functions of the beam coordinates, θ_o and φ_o, the coordinates of the field point, θ and φ, and the location of the element in the array. (See Appendix A for derivation of f, f_1, and f_2.)

IV. Computer Simulation

The design of the array described in Chapter III was accomplished using six computer programs written for the CDC 6600 computer using the Fortran Extended compiler. (See Appendix B for program listings.)

Program Pattern

Program Pattern is a program which computes the field contribution of each element in the array at the specified field point, sums the individual contributions and prints the value in tabular form. Field calculations are performed in a specified φ-plane for one degree increments of θ from $\theta = 0^\circ$ to $\theta = 90^\circ$. The program, given the number of elements around the base of the array and their spacing and the ratio of the height of the array to the radius of its base, will also calculate and store for later use the essential geometry for the field calculations (i.e., the number of elements in each circular subarray, the arc length between adjacent subarrays, total arc length of the array from base to top, and total number of elements in the array). The program calculates the field by solving equation (26) using as input variables the number of elements in the base of the array, the horizontal spacing of the array elements, the φ-coordinate of the field point, the angular coordinates of the beam, the ratio of the height of the array to the radius of the base, and the value of θ at which it is desired to begin calculations. This last input was provided in the event that only a particular region of space is of interest. Output from the program consists of a listing of the geometry of the array, a tabular listing of θ, E_θ, $|E_\theta|$, and the phase of E_θ, and the θ coordinate and magnitude of the maximum value of $|E_\theta|$. In addition, the listing of θ and E_θ is reproduced on punched cards for use with a plotting routine for graphical display of the array pattern as a function of θ.

Program Two

Program Two is a modification of Program Pattern which calculates the field for 0.1° increments of θ starting at the center of the main beam. This

program was provided to examine the field pattern in the immediate vicinity of the main beam in more detail.

Programs Three and Four

Programs Three and Four are modifications of Programs Pattern and Two respectively which compute the variation in the field pattern in the φ-direction. While the first two programs hold φ constant while θ is varied, the latter two hold θ constant while varying φ.

Program Beam

The fifth program in the series is still another modification of Program Pattern which calculates the magnitude and phase of the field at the center of the main beam as it is scanned from $\theta = 0^\circ$ to $\theta = 90^\circ$ in one degree steps. This program, like the first four, provides the output data in punched card format for use in the standard plot subroutines available with the CDC 6600.

Program Optimum

The sixth and last program is an optimization routine designed to find the ratio of the height to base radius of the array which yields the most nearly uniform gain as the beam is scanned from $\theta = 0^\circ$ to $\theta = 90^\circ$. Let H/R be the ratio of height to radius of the base and E_2/E_1 the ratio of the magnitudes of the beam at $\theta = 90^\circ$ and $\theta = 0^\circ$. The program takes an initial value of H/R and calculates E_2/E_1 for this configuration. H/R is then increased in steps of 0.1 until the ratio E_2/E_1 decreases. H/R is then decreased in steps of 0.01 until the ratio again begins to decrease. This procedure is continued until the optimum value of H/R is obtained. This ratio is then printed along with the geometry of the array. The design resulting from this program is then analyzed using the first five programs described above.

V. Results, Conclusions, and Recommendations

Results

An analysis of the output of Program Pattern, executed for an hemispherical array revealed that an array with the basic geometry described in Chapter III would indeed yield improved performance over a planar array at large scan angles. Figure 10 is a computerized plot of the magnitude of the E-field versus scan angle for an hemispherical array. It should be noted that this array pattern does not have the sharp drop in field intensity as the beam is scanned past 60° which is found in the planar array. With the fact established that an array with this shape could be made to scan through the range required, Program Optimum was executed and an optimum ratio of height of the array to the radius of its base was computed to be 2.49. It was also found during preliminary runs of Program Pattern that an array with 128 elements in the base ring would yield a beamwidth less than two degrees. These results fixed the geometry of the array entirely. Figure 11 is a scale drawing of the array design which is analyzed in some detail in the paragraphs to follow.

The execution of Program Beam revealed that the gain of the array is linear within 2 db as the beam is scanned from the vertical to the horizon (see Figs. 12 and 13). The minimum gain of the array referenced to a single radiating aperture oriented normal to the direction of the beam is approximately 29 db at $\theta = 90^\circ$.

Analysis of the results of Program Two revealed that the beamwidth in the θ-direction is not only less than 2° throughout the range of scan but actually decreases as the scan angle increases to a value of approximately 80°. Figures 14 and 15 show the variation in beamwidth in the two directions with scan angle. The increase in beamwidth at scan angles greater than 80° can be attributed to edge effects as the beam approaches the lower edge of the array. It should be noted, however, that the maximum beamwidth of approximately 1.5°

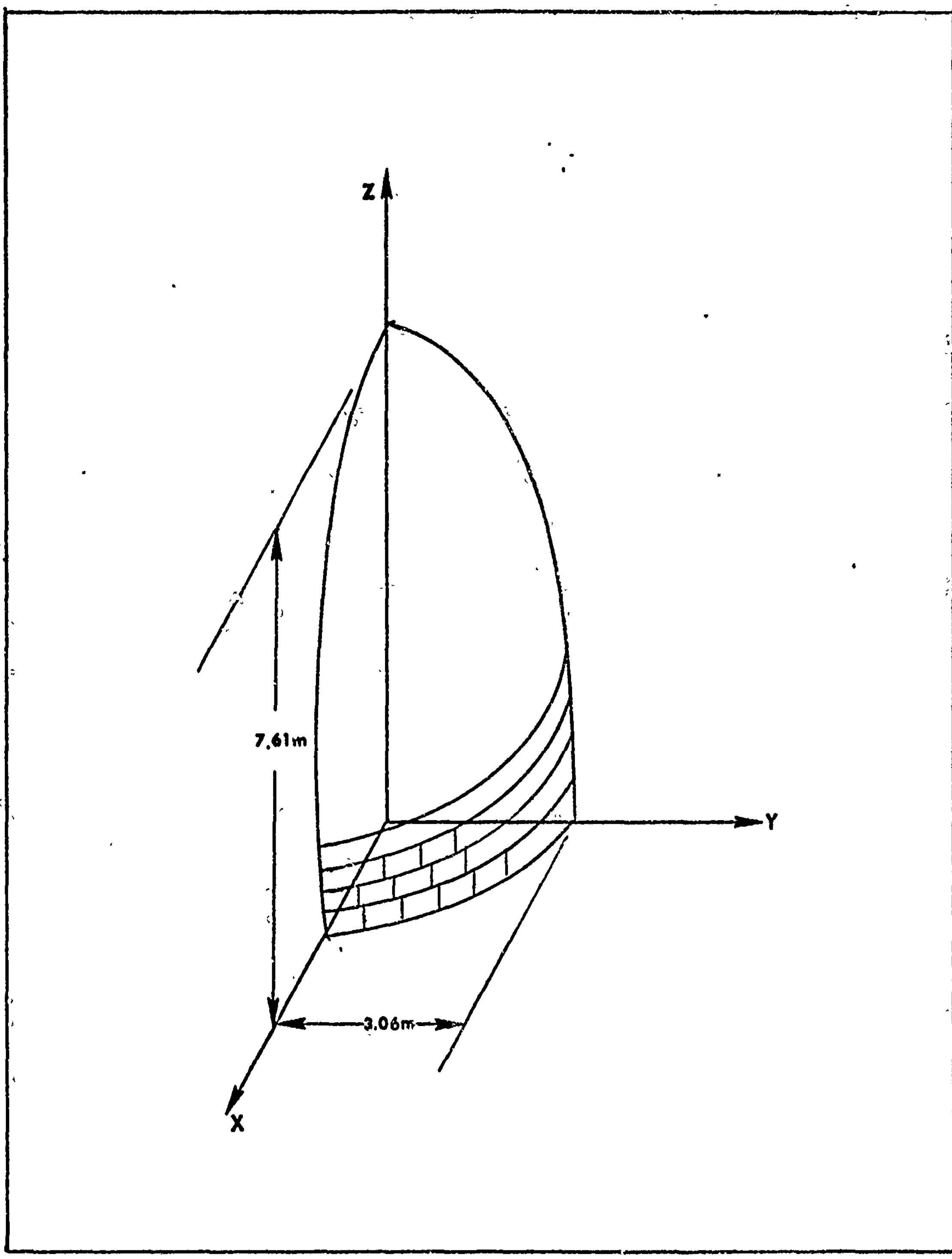

Fig. 10. Beehive Array Resulting from Execution of Program Optimum (See Appendix C for Complete Specifications)

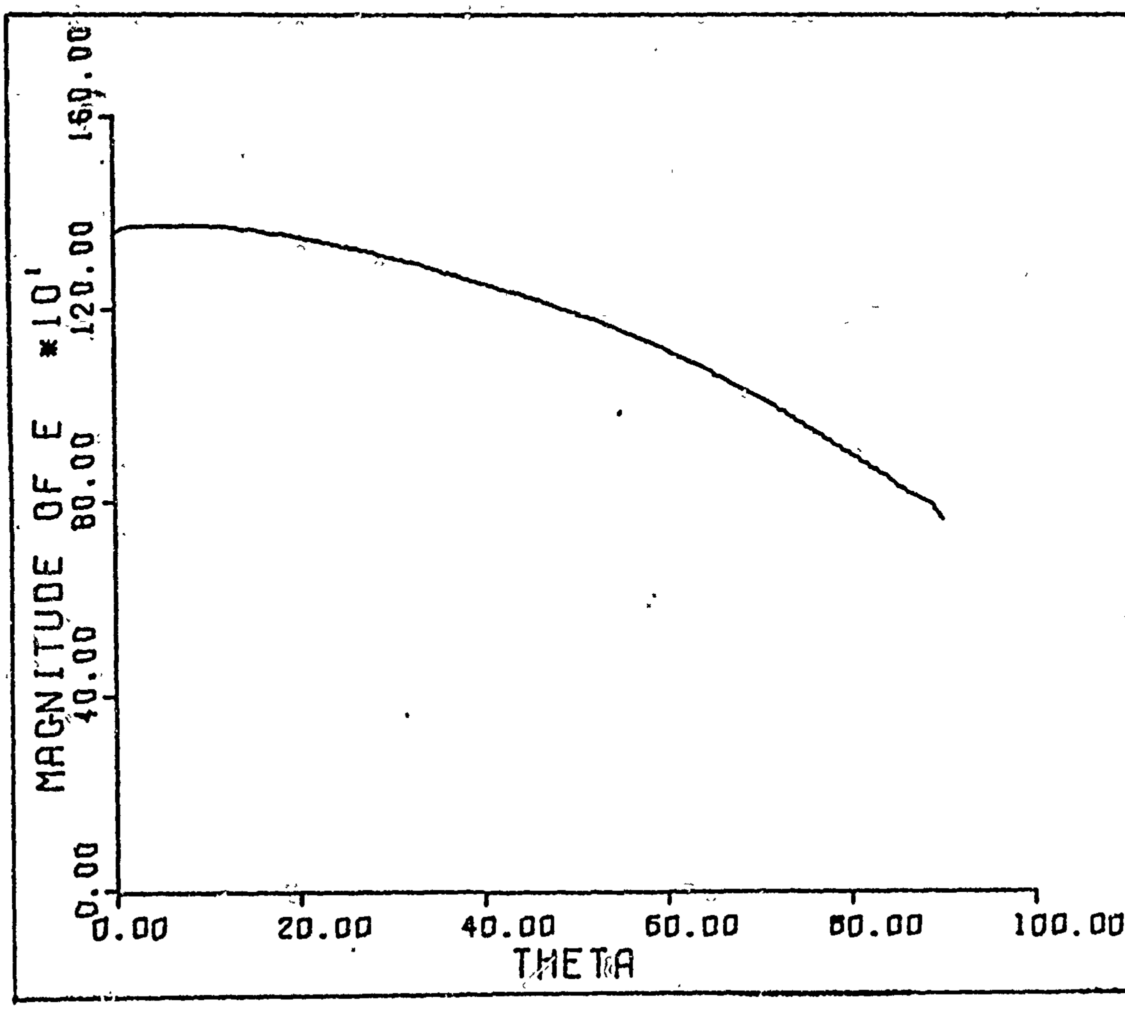

Fig. 11. $|E_\theta|$ vs θ for Hemispheric Array

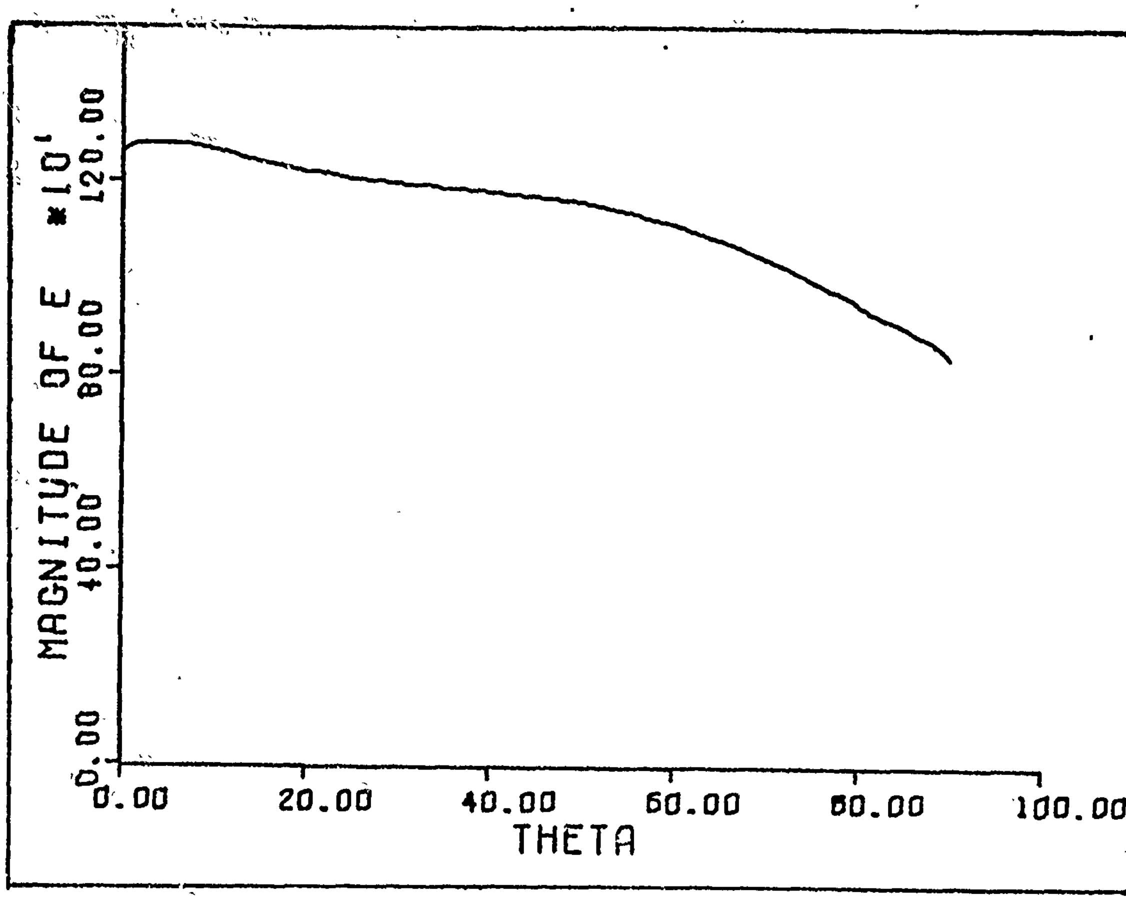

Fig. 12. Field Intensity vs Scan Angle for Optimal Array

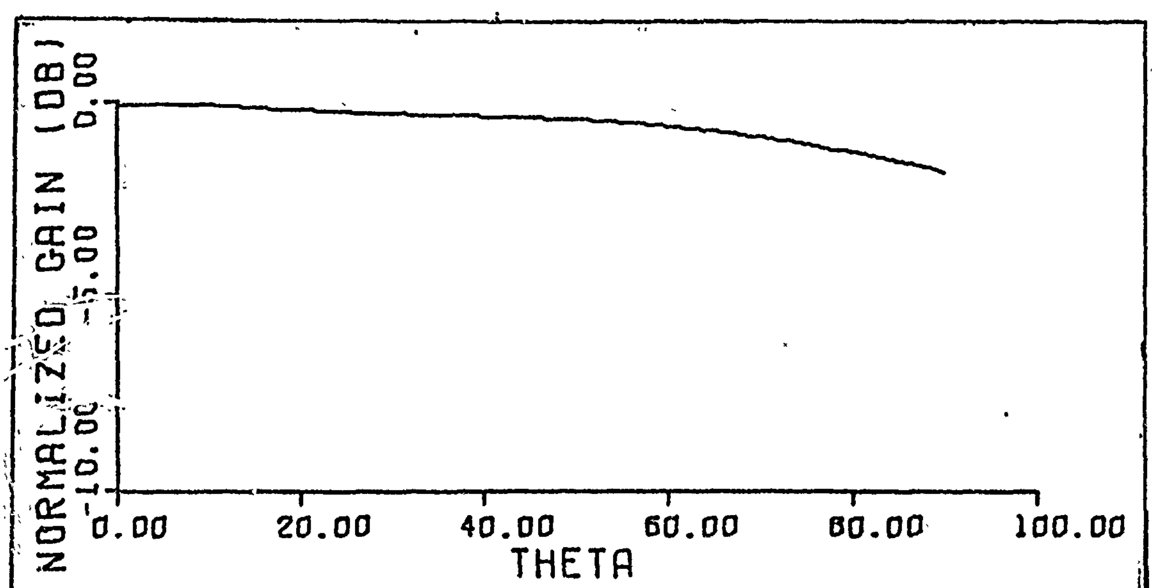

Fig. 13. Gain vs Scan Angle of Optimum Array Design

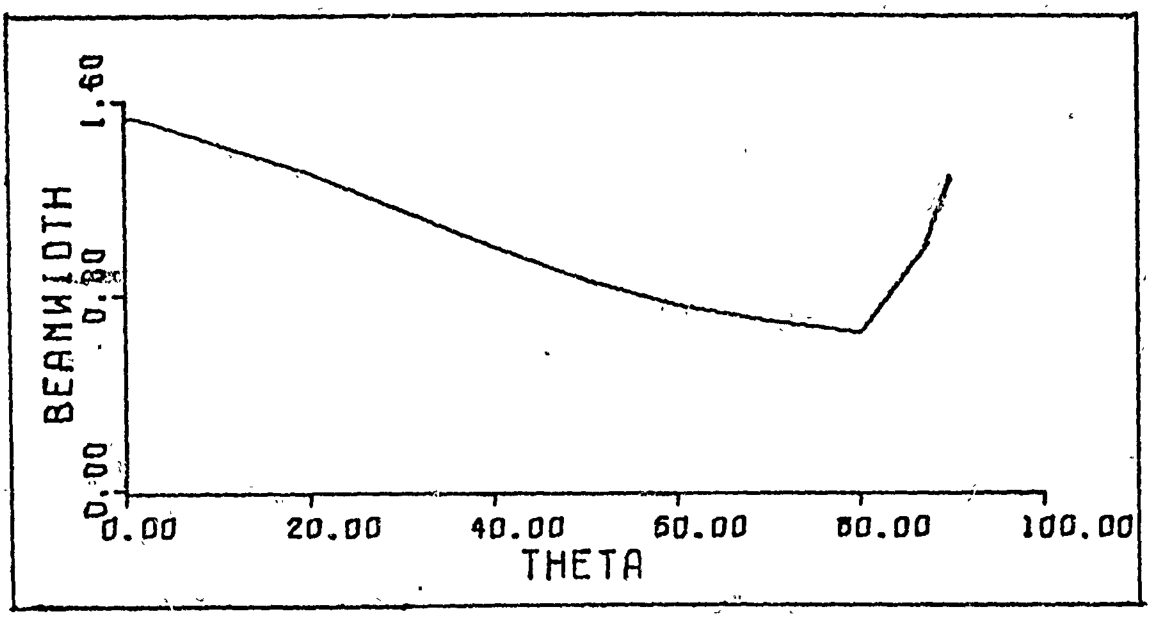

Fig. 14. θ Beamwidth vs Scan Angle

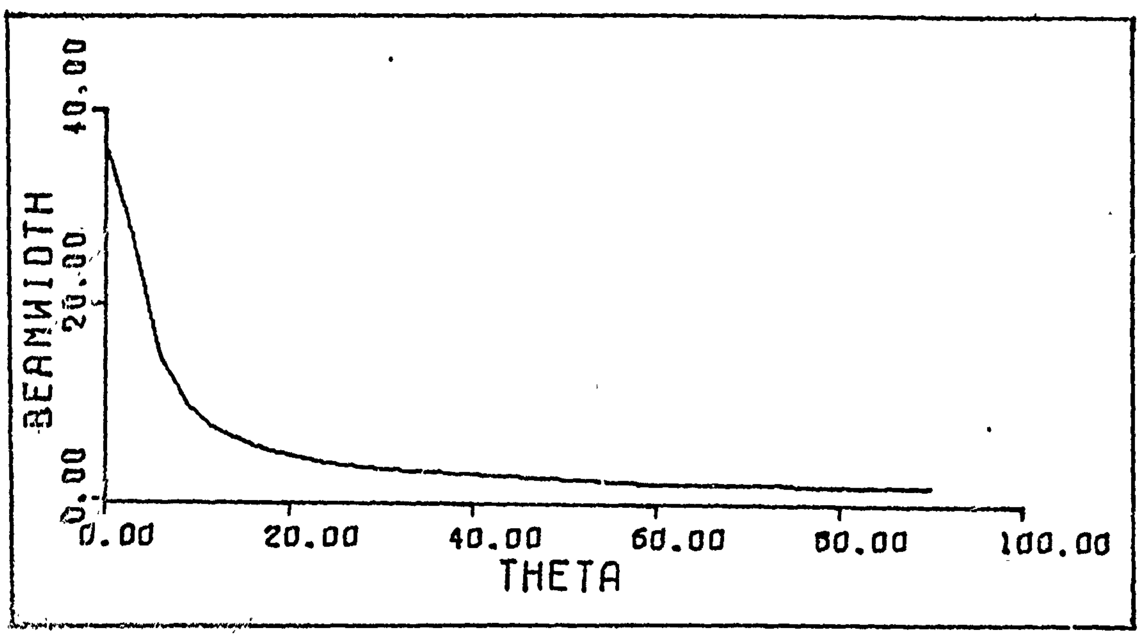

Fig. 15. φ Beamwidth vs Scan Angle

occurs not at the horizon but at a scan angle of 0°. Program Four was executed for various scan angles to determine the beamwidth and sidelobe levels in the φ-direction. Study was concentrated mainly in the region near the pole of the array since the symmetry of the array indicated little change in φ characteristics of the beam with scan angle. It was noted that the beamwidth near the pole is very broad. However, this result is inconsequential since any value for a beamwidth in the φ-direction near $\theta = 0^\circ$ is meaningless.

Investigation of the field pattern on the opposite side of the array from the main beam revealed the presence of two small lobes located 136° to either side of the beam when the beam is scanned to $\theta = 60^\circ$. (See Fig. 16.) It is reasonable to assume that these lobes are present at other scan angles also. Further investigation of the field pattern in this vicinity revealed that the magnitude of this lobe is less than 1 percent of the magnitude of the main beam and should be of little significance.

Results of the execution of Programs Pattern, Two, Three, and Four for various beam positions revealed that the maximum sidelobe level of the array is -9 db, which is somewhat higher than the sidelobe level of a planar array with uniform spacing and excitation. However, sidelobe levels decreased as the scan angle is increased as shown in Figs. 17 through 23.

It was also discovered during analysis of the output of Program Two that the calculated beam position varies from the specified position by approximately 0.4° in the θ-direction at scan angles greater than approximately 80°. However, this should be of no significant consequence since an offset can be added to the desired scan angle to compensate for this inaccuracy.

Conclusions

It has been stated in this study that a planar array is unsuited to the task of satellite tracking due to the undesirable characteristics of the array at large scan angles. It is shown that the task can be accomplished using a non-planar array of 3796 rectangular apertures arranged in the shape of a beehive with a height equal to 2.49 times the radius of its base. The resulting phased

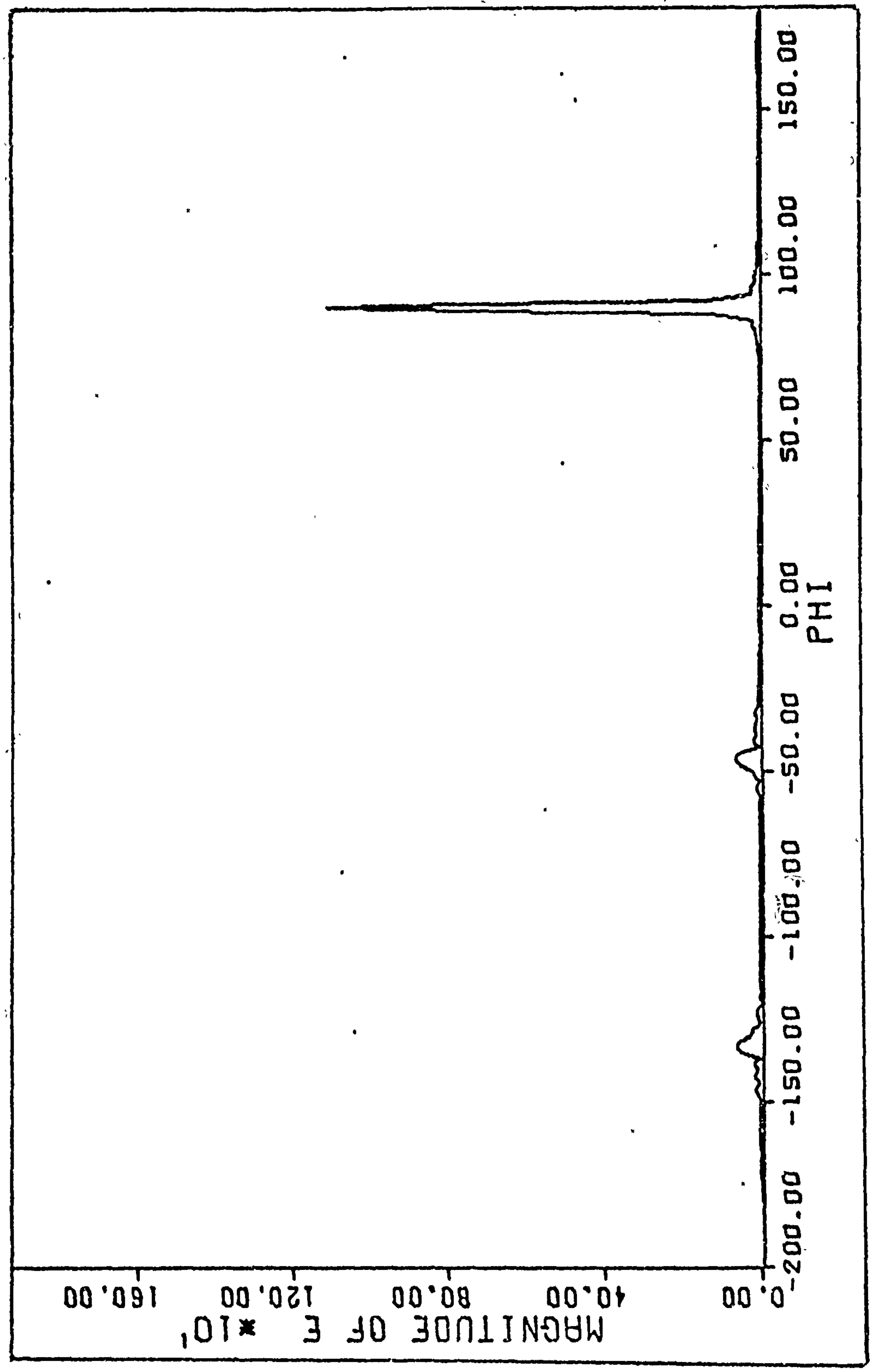

Fig. 16. Field Intensity vs φ for Optimum Array Design

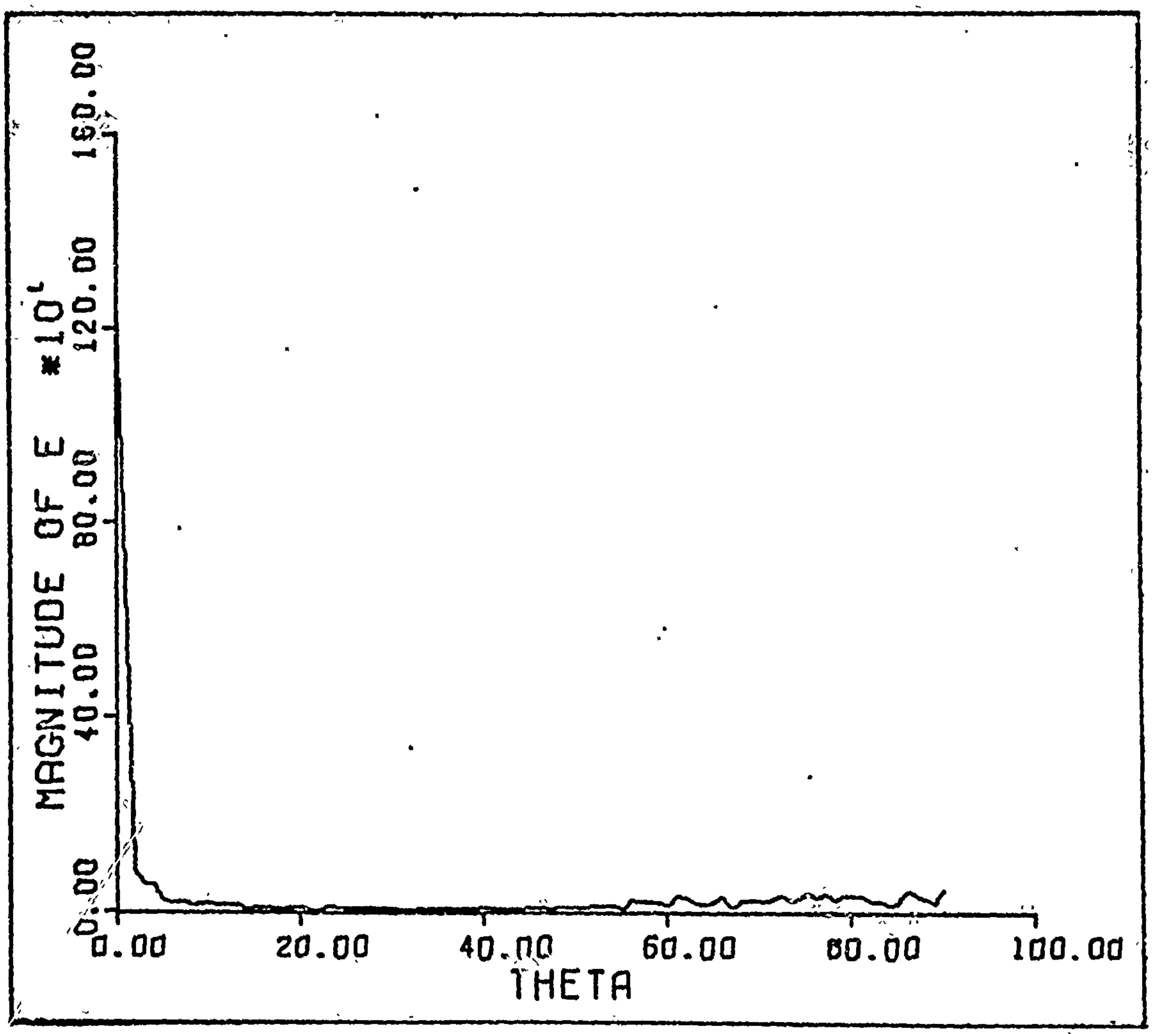

Fig. 17. Field Intensity vs θ Beam at 0°

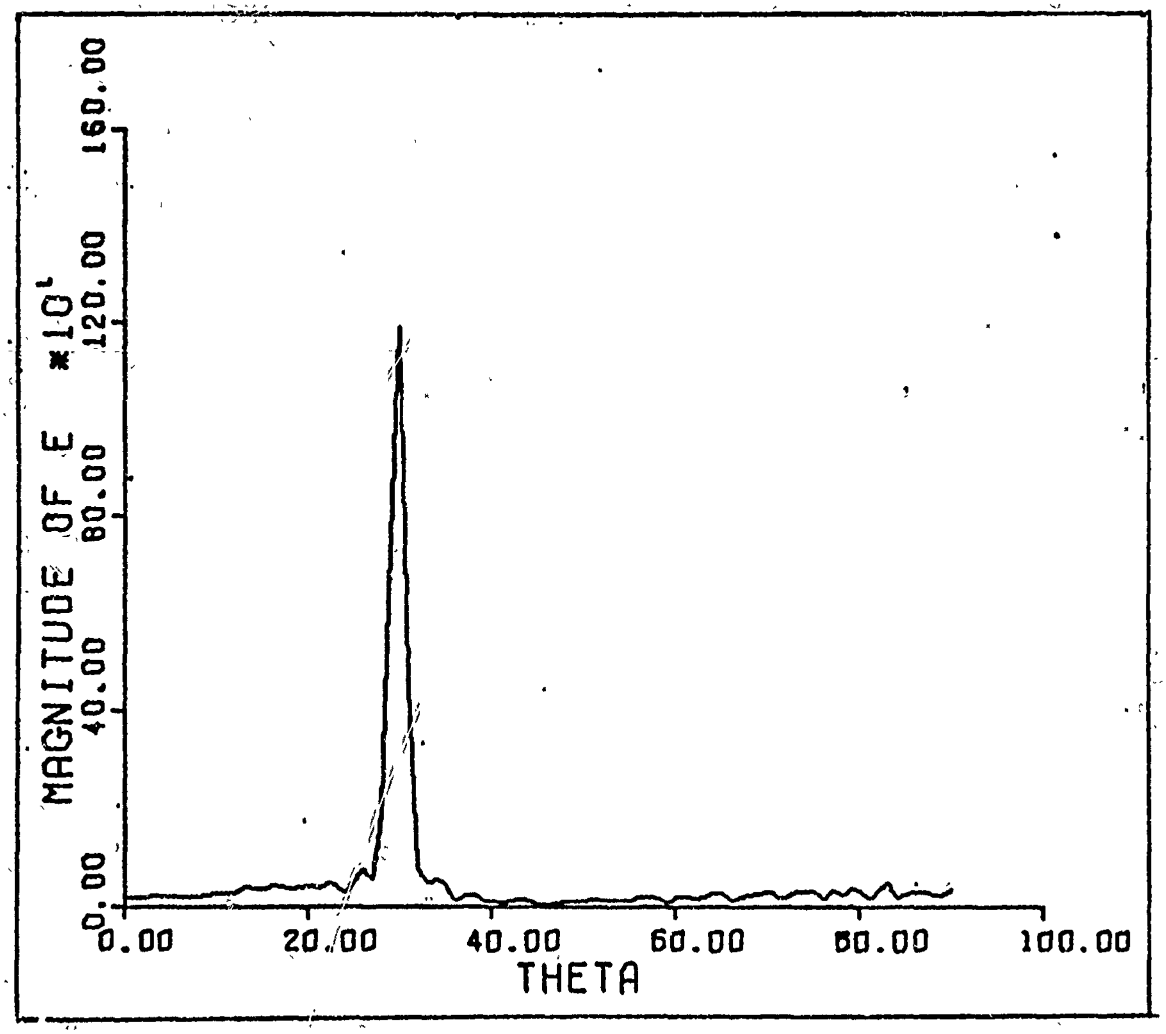

Fig. 18. Field Intensity vs θ. Beam at 30°

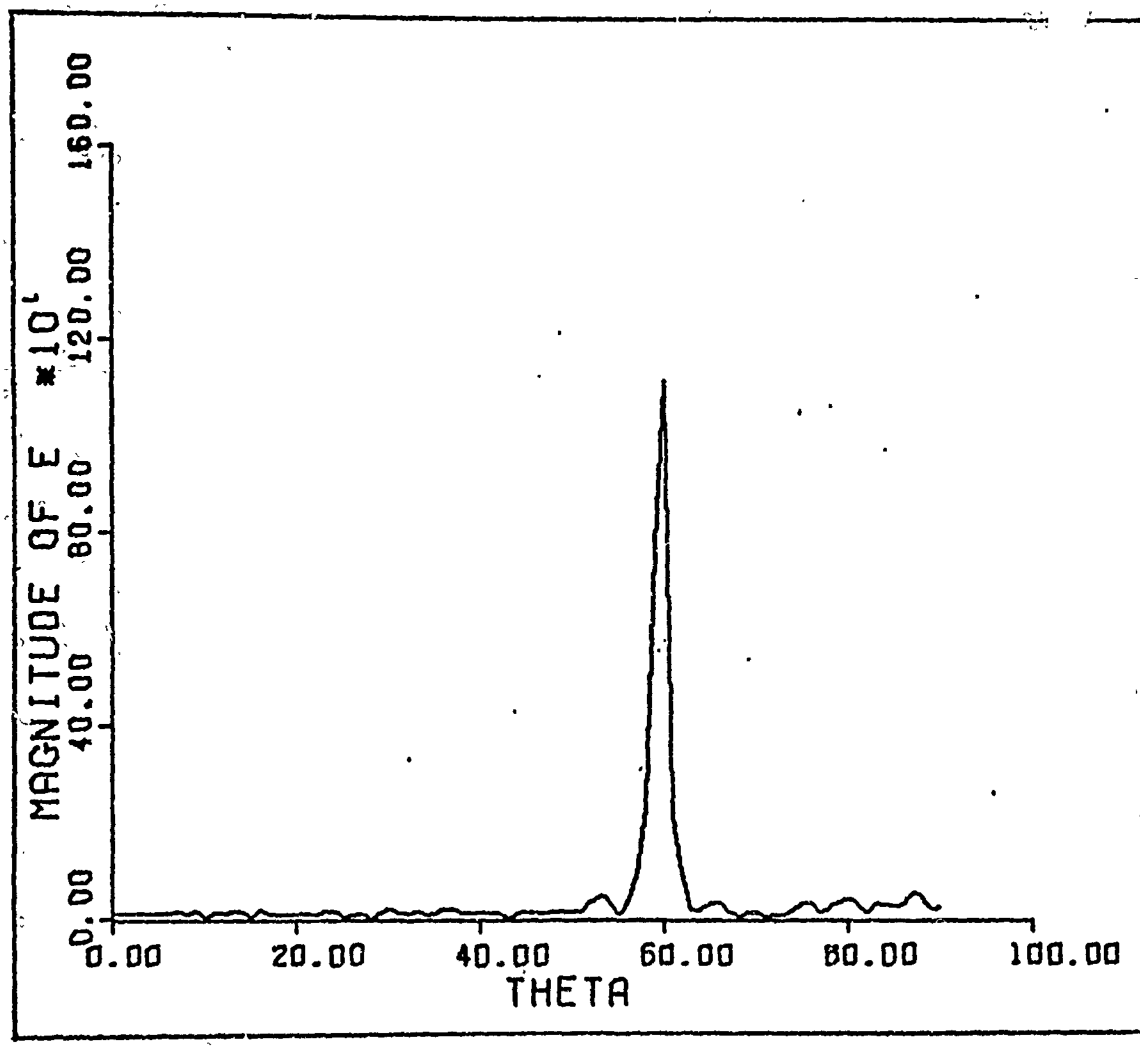

Fig. 19. Field Intensity vs θ. Beam at 60°

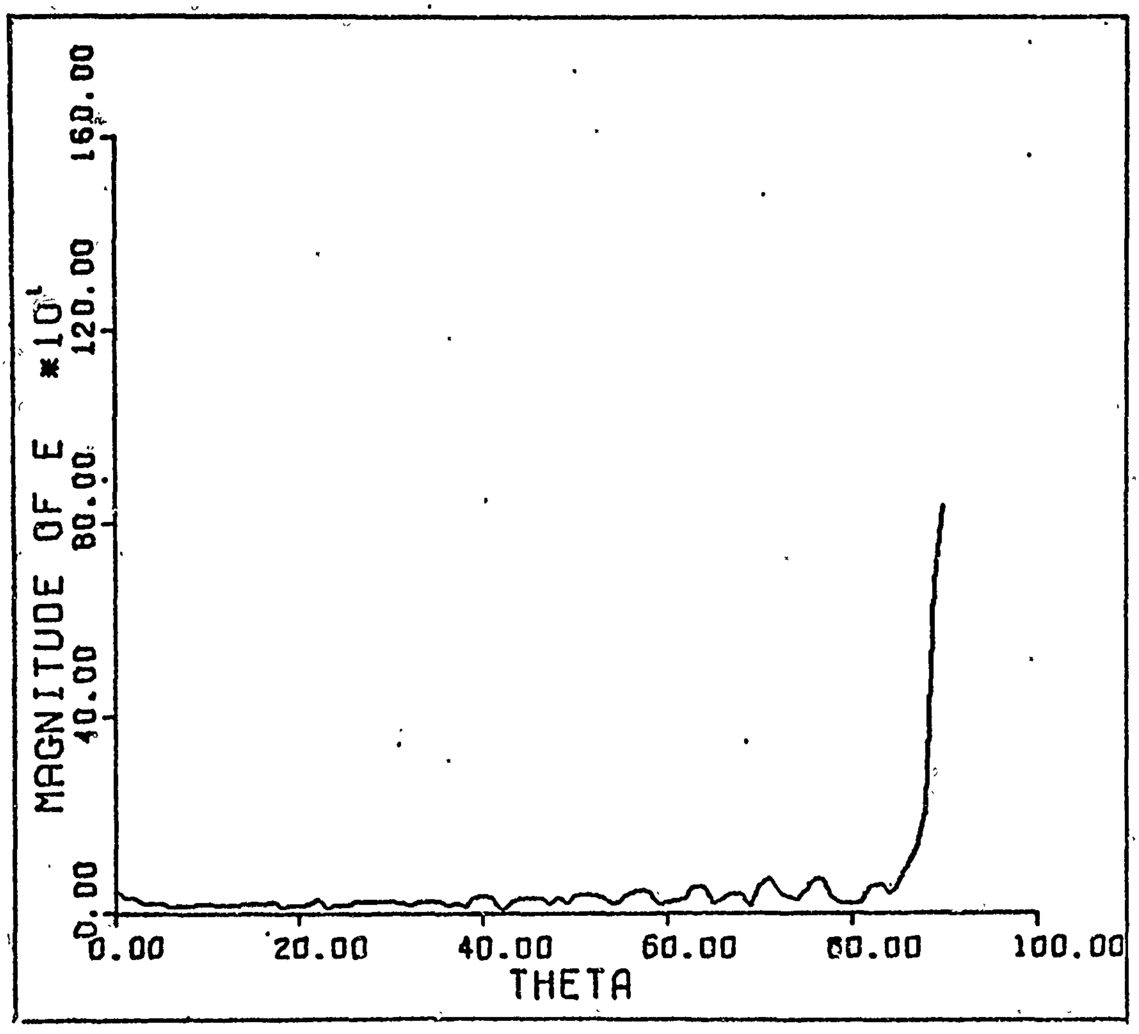

Fig. 20. Field Intensity vs θ. Beam at 90°

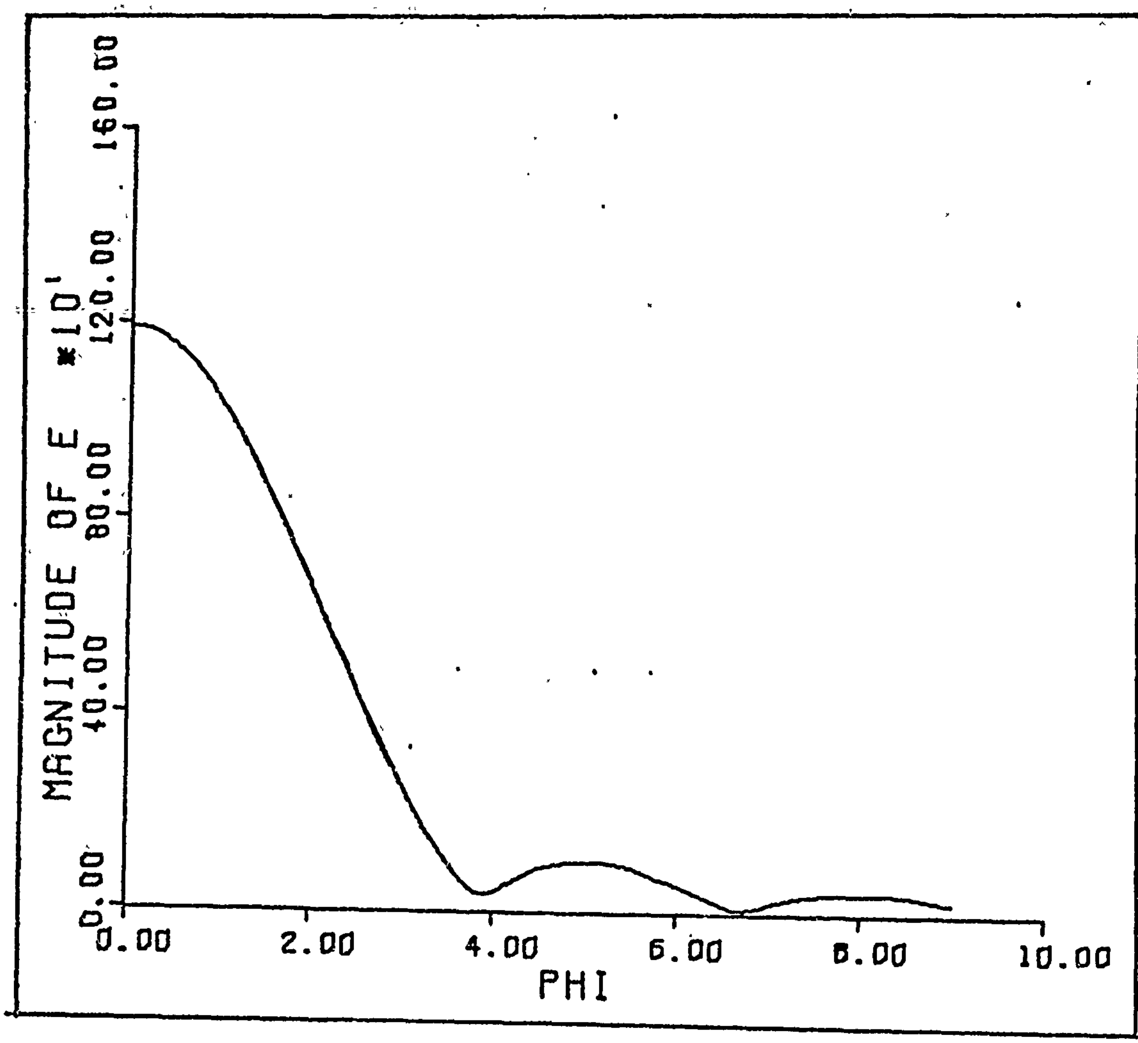

Fig. 21. Field Intensity vs φ. Beam at 30°

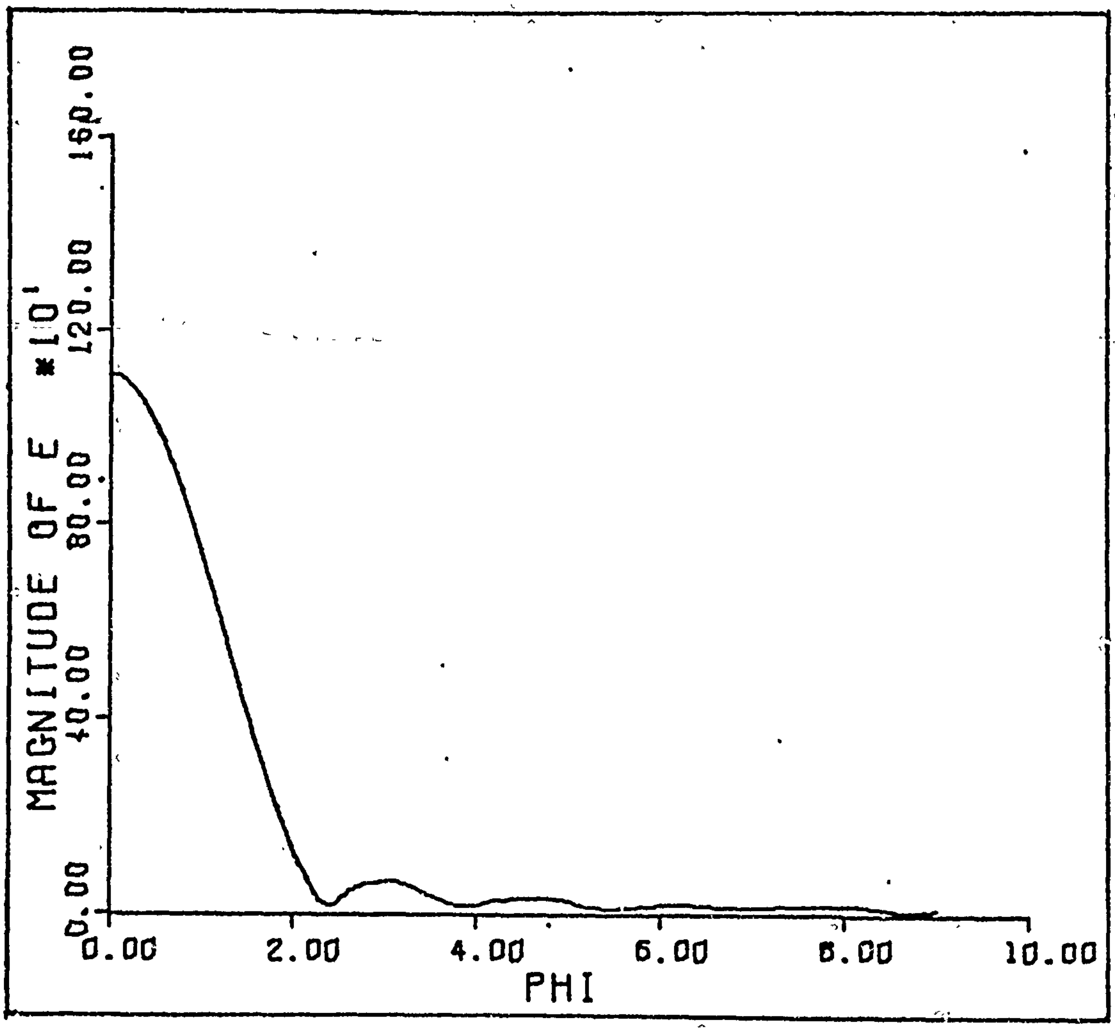

Fig. 22. Field Intensity vs φ. Beam at 60°

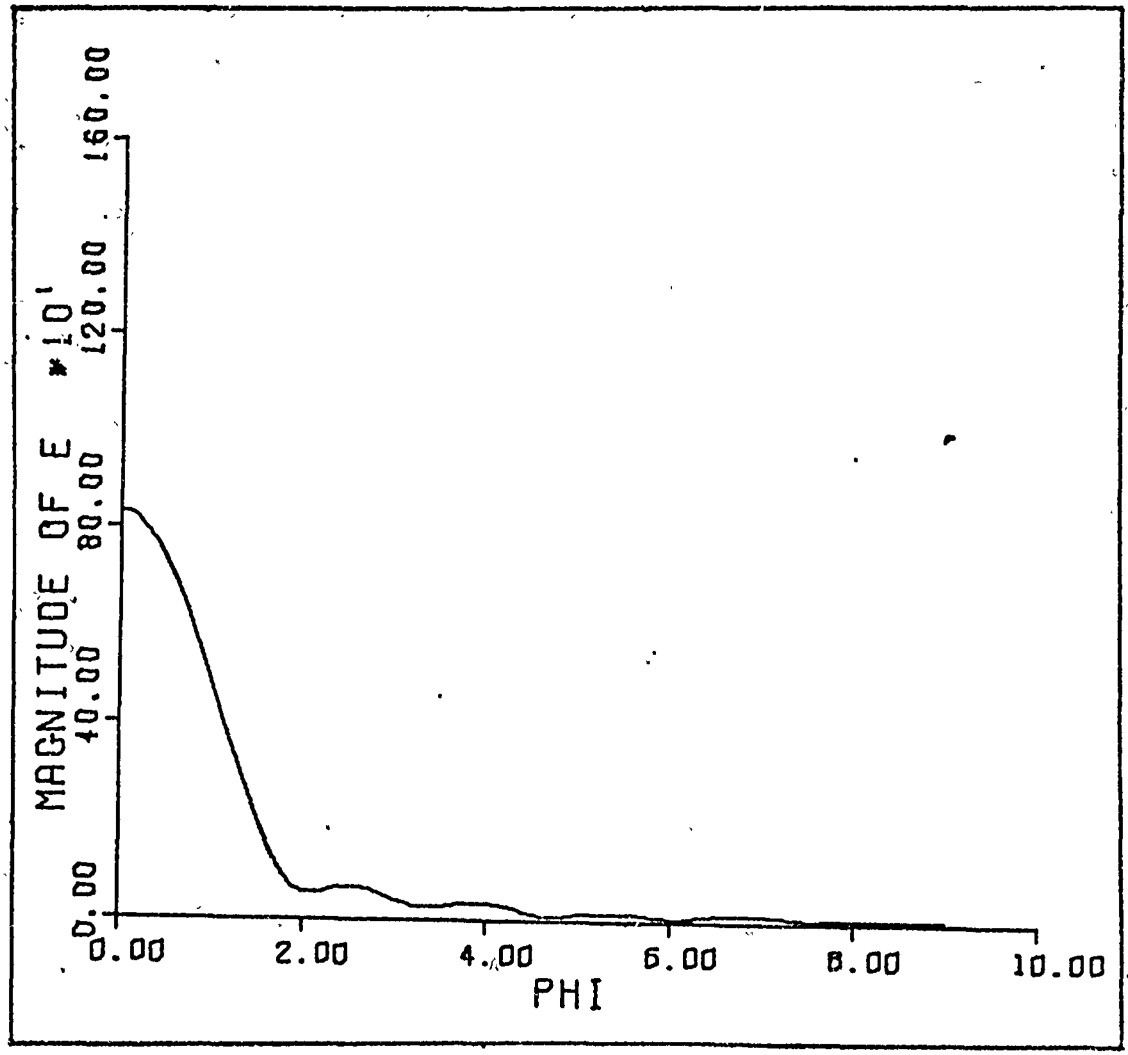

Fig. 23. Field Intensity vs φ. Beam at 90°

array has a very high gain with relatively low sidelobes and a narrow beam. The gain is very nearly uniform throughout the range of scan required. While the maximum sidelobe level has deteriorated from that of the planar array, there are techniques such as non-uniform excitation and spacing available to reduce the sidelobe levels to acceptable values.

Recommendations

It is recommended that further studies of the beehive array be conducted to determine the effects of mutual coupling and the possibilities of using the technique of iris loading of the apertures to minimize these effects (Ref 6). The effect of non-uniform spacing and excitation of the elements in the array could be studied to determine whether performance of the array is improved (Ref 4).

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Appendix A

Derivation of Equations For Field Calculations

The far field of an elementary antenna element at any point in space may be expressed as

$$E_\theta = f(\theta,\varphi)\frac{e^{-j k_o d}}{d} e^{j\alpha} \tag{27}$$

where d is the distance from the element to the field point, $f(\theta,\varphi)$ is determined by the orientation and excitation of the element, and α is the phase relative to some chosen reference.

For the geometry specified in Chapter III,

$$d = [X^2 + R^2 + Z_1^2 - 2XR\sin\theta \operatorname{vos}(\varphi-\varphi_1) - 2RZ\cos\theta]^{\frac{1}{2}} \tag{28}$$

where X, R, Z_1, θ, φ, and φ_1 are as shown in Fig. 24.

Reference Fig. 25. If the arc length along the ellipse described by the equation

$$R^2 = X^2 + Z^2/E \tag{29}$$

is very small, it may be approximated by a straight line chord for the purpose of calculating the value of X. By the Pythagorean Theorem

$$Z_1^2 + (R - X_1)^2 = (\lambda/2)^2. \tag{30}$$

Substitution for Z from Equation (29) yields

$$X = \frac{2R - [\lambda^2(1-E) + 4E^2R^2]^{\frac{1}{2}}}{2(1-E)} \tag{31}$$

$$Z = [(R^2 - X^2)E]^{\frac{1}{2}} \tag{32}$$

and

$$\theta_1 = \tan^{-1}(Z/X). \tag{33}$$

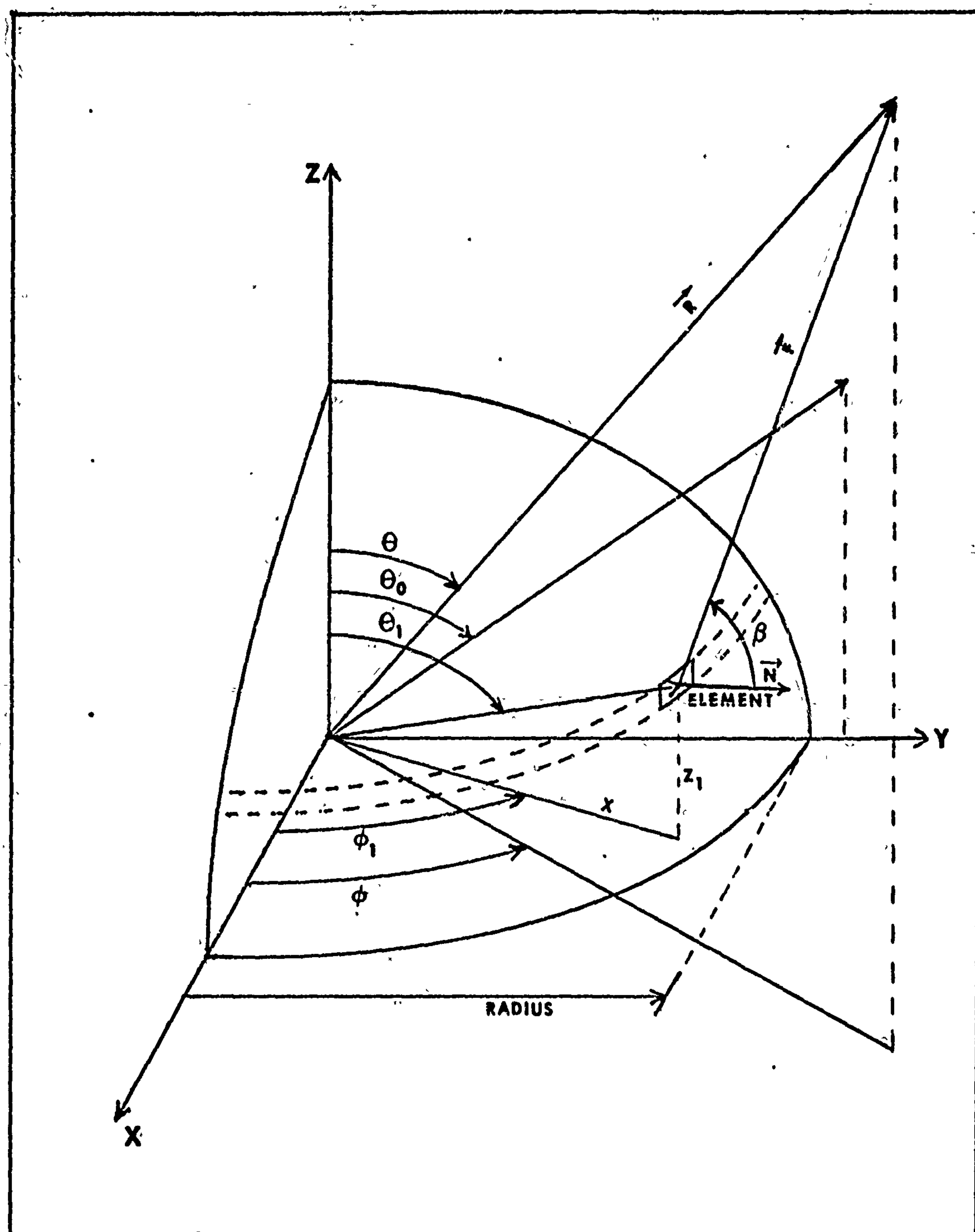

Fig. 24. Geometry Used to Calculate Field Pattern of Array

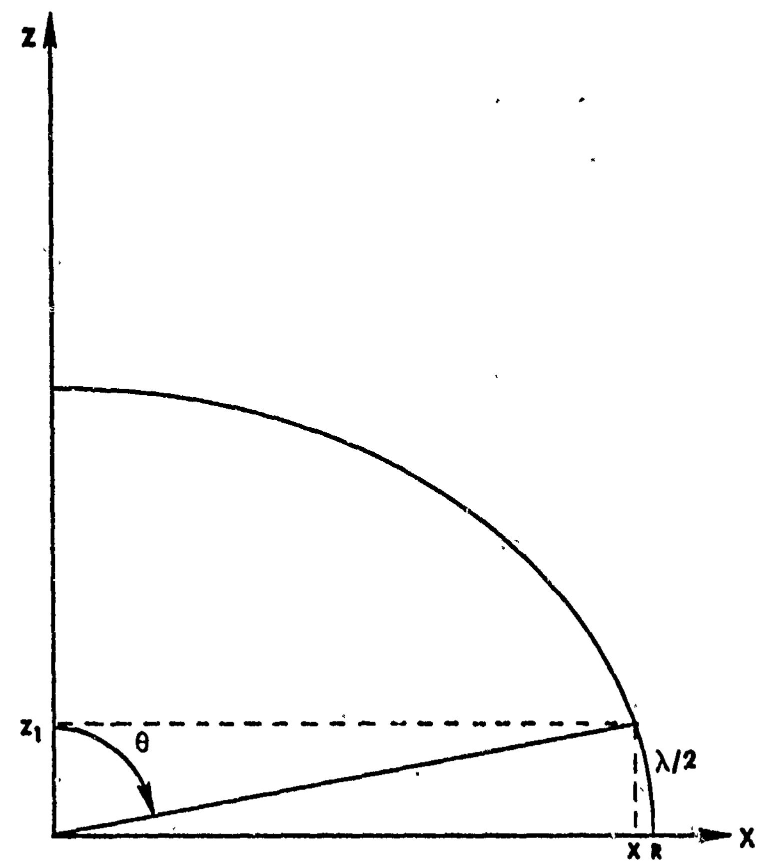

Fig. 25. Geometry Used to Calculate Radius of Second Circle of Elements

For each succeeding circle around the array, the radius, X, (reference Fig. 26) may be calculated according to the equation

$$X_1 = X - (\lambda/2)\cos\alpha, \tag{34}$$

where

$$\alpha = \pi - \tan^{-1}(dz/dx)$$

$$= \pi - \tan^{-1}\left[\frac{-X}{[(R^2 - X^2)E]^{\frac{1}{2}}}\right]. \tag{35}$$

Arc length along the ellipse described by Equation (29) is given by

$$S = \int_{X}^{X_1} [1 + (dy/dx)^2]^{\frac{1}{2}}\, dx$$

$$= \int_{X}^{X_1} \left(\frac{R^2 - X^2(1 - E)}{R^2 - X^2}\right)^{\frac{1}{2}} dx. \tag{36}$$

Reference Fig. 24

$$\beta = \cos^{-1}\left[\frac{\overline{F}\bullet\overline{N}}{|F|\;|N|}\right] \tag{37}$$

$$\overline{F} = \overline{R} - \overline{P}$$

$$\overline{R} = R\,[\overline{a}_x \sin\theta\cos\varphi + \overline{a}_y \sin\theta\sin\varphi + \overline{a}_z \cos\theta] \tag{38}$$

$$\overline{P} = X[\,\overline{a}_x \cos\varphi_1 + \overline{a}_y \sin\varphi_1\,] + \overline{a}_z Z_1 \tag{39}$$

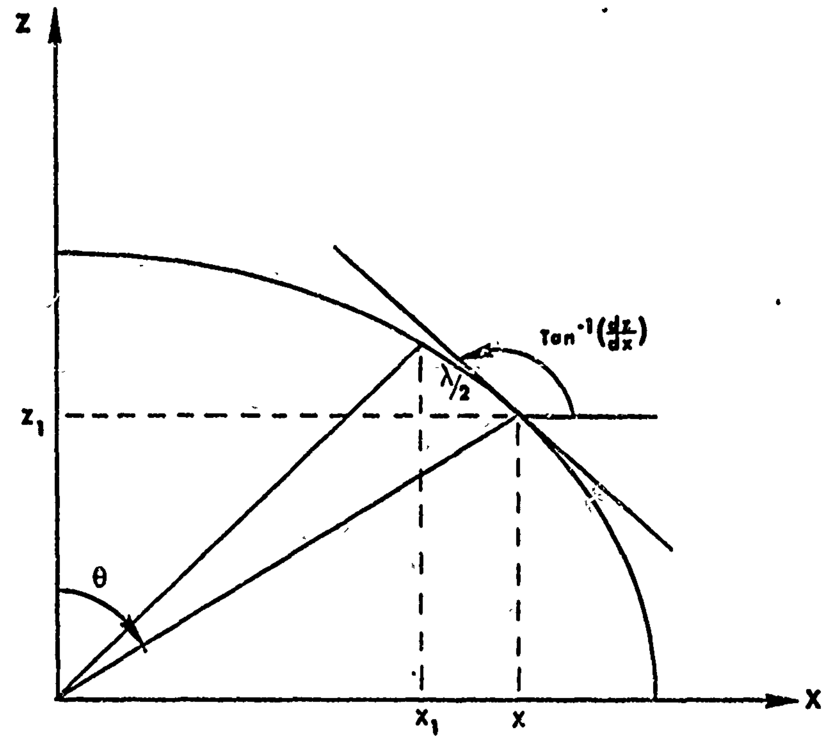

Fig. 26. Geometry Used to Calculate Third and Subsequent Circles

Appendix B

Computer Programs

```
      PROGRAM PATTERN(INPUT,OUTPUT,PUNCH)
      COMPLEX RANGE,PHASE,A,W,Z
      REAL LAMBDA,K
       DIMENSION A(91),AMAG(91),APHASE(91),NP(500),ARC(500)
      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
  100 FORMAT(2I5,3F5.0,F5.3,F3.0)
  700 FORMAT(1H ,F6.1,3F15.5,F6.0)
  800 FORMAT(1H1,*SPACING *,I3,*/16 WAVELENGTH*,4X,*RADIUS*,F6.2,* METER
     CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F6.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
     C,F5.3,* OF BASE*)
  801 FORMAT(1H0,*MAXIMUM *,F15.5,* AT *,F3.0,*DEGREES*,/)
  803 FORMAT(1H ,*BEAM COORDINATES--THETA= *,F3.0,2X,*PHI= *,F3.0,/)
  804 FORMAT(1H ,I6,9X,I5)
  805 FORMAT(1H ,*##### NUMBER OF ELEMENTS IN ARRAY #####*,/ ,* CIRCLE N
     CUMBER*,4X,*NUMBER OF ELEMENTS*)
  806 FORMAT(1H0,*TOTAL NUMBER OF ELEMENTS IN ARRAY--*,I8)
  807 FORMAT(F6.1,3F15.5,F6.0)
  808 FORMAT(F5.3,3F15.5,2F3.0)
CCCC
CCCC  DEFINES CONSTANTS TO BE USED IN PROGRAM
CCCC
      FREQ=2.*10.**9.
      PSI=0.
      PI=4.*ATAN(1.0)
      RAD=360./(2.*PI)
      LAMBDA=3.*10.**8./FREQ
      K=2.*PI*FREQ/(3.*10.**8.)
      GFREQ=FREQ/(10.**9.)
CCCC
CCCC  BEGIN CALCULATIONS
CCCC  N=NUMBER OF ELEMENTS AROUND BASE OF ARRAY
CCCC  B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
CCCC  THETAO AND PHIP ARE BEAM COORDINATES IN DEGREES
CCCC  E1=RATIO OF HEIGHT OF ARRAY TO RADIUS OF BASE
CCCC  THETA=START ANGLE FOR CALCULATIONS
CCCC
```

```
CCCC
  202 READ 100,N,L,B,THETA0,PHIP,E1,THETA
      IF(N)503,503,502
CCCC
CCCC  THROUGH 702 - INITIALIZES ARRAYS
CCCC
  502 SUMARC=0.
      DO 701 I=1,500
      ARC(I)=0.
  701 NP(I)=0
      DO 702 I=1,91
      A(I)=(0.,0.)
  702 AMAG(I)=APHASE(I)=0.
CCCC
CCCC  CALCULATES REMAINING CONSTANTS DEPENDENT ON INPUT VARIABLES
CCCC
      E=E1**2
      M1=1
      NP(1)=N
      PHIO=PI*B/N+PI/2.
      PH=PHIO*RAD
      PHI1=PI/2.-PHIP/RAD
      RADIUS=N*LAMBDA*L/(32.*PI)
      H=RADIUS/1000.
      X=H/2.
      R=200.*RADIUS
CCCC
CCCC  CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
CCCC
      TARC=0.
      DO 110 I=1,1000
      TARC=TARC+H*F(X)
  110 X=X+H
      PRINT 800,L,RADIUS,GFREQ,PH,E1
      PRINT 803,THETA0,PHIP
      PUNCH 808,E1,N,THETA0,PH
```

```
CCCC
CCCC  THROUGH 205 - CALCULATES NUMBER OF ELEMENTS IN EACH SUB ARRAY
CCCC    AND ARC LENGTH BETWEEN THEM
CCCC
      NUM=N
      MM=2
  203 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
     CE))
      X2=RADIUS
  207 NP(MM)=PI*X/LAMBDA
      NP(MM)=2*NP(MM)
      IF(MM-1)240,240,239
  239 IF(NP(MM)-NP(MM-1))240,241,241
  241 NP(MM)=NP(MM-1)-2
  240 NUM=NUM+NP(MM)
      X=NP(MM)*LAMBDA/(2.*PI)
      H=(X2-X)/100.
      Y=X+H/2.
      DO 115 II=1,100
      ARC(MM)=ARC(MM)+H*F(Y)
  115 Y=Y+H
      SUMARC=SUMARC+ARC(MM)
      IF(TARC-SUMARC-LAMBDA/2.)205,205,204
  204 ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
      X1=X-(LAMBDA/2.)*COS(ALPHA)
      X2=X
      X=X1
      MM=MM+1
      GO TO 207
  205 CONTINUE
      MMAX=MM
      MMA=MM+1
CCCC
CCCC  DO 400 LOOP CALCULATES FIELD FOR THETA=M DEGREES
CCCC
```

```
      DO 400 M=1,91
      S7=SIN(THETA0/RAD)
      C7=COS(THETA0/RAD)
      S5=SIN(PHI0)
      C5=COS(PHI0)
      C4=COS(THETA/RAD)
      S4=SIN(THETA/RAD)
      X=RADIUS
      Z1=0.
      A(M)=(0.,0.)
      ALPHA=PI/2.
      THETA1=PI/2.
CCCC
CCCC  DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
CCCC
      DO 250 MM=1,MMAX
      S1=SIN(ALPHA)
      C1=COS(ALPHA)
      S2=SIN(THETA1)
      C2=COS(THETA1)
      N=NP(MM)
CCCC
CCCC  DO 200 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
CCCC
      DO 200 I=1,N
      PHI=2.*PI*(I-1)/N
      S3=SIN(PHI)
      C3=COS(PHI)
      S6=SIN(PHI1-PHI)
      C6=COS(PHI1-PHI)
      S8=SIN(PHI0-PHI)
      C8=COS(PHI0-PHI)
```

```
      PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
      DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
      BETA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)
      IF(BETA)231,208,231
  231 IF(BETA-PI/2.)209,232,232
  232 ELTFAC=0.
      GO TO 211
  208 ELTFAC=1.
      GO TO 211
  209 ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
  211 W=(0.,-1.)*DIST
      Z=(0.,1.)*PSI
      RANGE=K*W
      PHASE=CEXP(Z)
  200 A(M)=A(M)+CEXP(RANGE)*PHASE*ELTFAC
      X=NP(MM+1)*LAMBDA/(2.*PI)
      Z1=SQRT((RADIUS**2.-X**2.)*E)
      ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
  250 THETA1=ATAN2(X,Z1)
      AMAG(M)=CABS(A(M))
      APHASE(M)=ATAN2(AIMAG(A(M)),REAL(A(M)))*RAD
      PRINT 700,THETA,A(M),AMAG(M),APHASE(M)
      PUNCH 807,THETA,A(M),AMAG(M),APHASE(M)
  400 THETA=THETA+1.
CCCC
CCCC  THROUGH 402 - FINDS MAX FIELD AND ANGLE
CCCC
      D=AMAG(1)
      DO 300 J=2,91
  300 D=AMAX1(D,AMAG(J))
      DO 401 J=1,91
      JJ=J
      IF(AMAG(J)-D)401,402,401
  401 CONTINUE
  402 THETA=FLOAT(JJ-1)
      PPINT 801,D,THETA
```

```
      PRINT 805
      DO 900 I=1,MMAX
  900 PRINT 804,I,NP(I)
      PFINT 804,MMA,M1
      PRINT 806,NUM
      GO TO 202
  503 STOP
      END
```

```
      PROGRAM TWO(INPUT,OUTPUT,PUNCH)
      COMPLEX RANGE,PHASE,A,W,Z
      REAL LAMBDA,K
       DIMENSION A(91),AMAG(91),APHASE(91),NF(500),ARC(500)
      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
  100 FORMAT(2I5,3F5.0,F5.3,F3.0)
  700 FORMAT(1H ,F6.1,3F15.5,F6.0)
  800 FORMAT(1H1,*SPACING *,I3,*/16 WAVELENGTH*,4X,*RADIUS*,F6.2,* METER
     CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F6.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
     C,F5.3,* OF BASE*)
  801 FORMAT(1H0,*MAXIMUM *,F15.5,* AT *,F3.0,*DEGREES*,/)
  803 FORMAT(1H ,*BEAM COORDINATES--THETA= *,F3.0,2X,*PHI= *,F3.0,/)
  804 FORMAT(1H ,I6,9X,I5)
  805 FORMAT(1H ,*##### NUMBER OF ELEMENTS IN ARRAY #####*,/ ,* CIRCLE N
     CUMBER*,4X,*NUMBER OF ELEMENTS*)
  806 FORMAT(1H0,*TOTAL NUMBER OF ELEMENTS IN ARRAY--*,I8)
  807 FORMAT(F6.1,3F15.5,F6.0)
  808 FORMAT(F5.3,3F15.5,2F3.0)
CCCC
CCCC  DEFINES CONSTANTS TO BE USED IN PROGRAM
CCCC
      FREQ=2.*10.**9.
      PSI=0.
      PI=4.*ATAN(1.0)
      RAD=360./(2.*PI)
      LAMBDA=3.*10.**8./FREQ
      K=2.*PI*FREQ/(3.*10.**8.)
      GFREQ=FREQ/(10.**9.)
CCCC
CCCC  BEGIN CALCULATIONS
CCCC  N=NUMBER OF ELEMENTS AROUND BASE OF ARRAY
CCCC  B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
CCCC  THETA0 AND PHIP ARE BEAM COORDINATES IN DEGREES
CCCC  E1=RATIO OF HEIGHT OF ARRAY TO RADIUS OF BASE
CCCC  THETA=START ANGLE FOR CALCULATIONS
```

```
CCCC
  202 READ 100,N,L,B,THETA0,PHIP,E1,THETA
      IF(N)503,503,502
CCCC
CCCC  THROUGH 702 - INITIALIZES ARRAYS
CCCC
  502 SUMARC=0.
      DO 701 I=1,500
      ARC(I)=0.
  701 NP(I)=0
      DO 702 I=1,91
      A(I)=(0.,0.)
  702 AMAG(I)=APHASE(I)=0.
CCCC
CCCC  CALCULATES REMAINING CONSTANTS DEPENDENT ON INPUT VARIABLES
CCCC
      E=E1**2
      M1=1
      NP(1)=N
      PHIO=PI*B/N+PI/2.
      PH=PHIO*RAD
      PHI1=PI/2.-PHIP/RAD
      RADIUS=N*LAMBDA*L/(32.*PI)
      H=RADIUS/1000.
      X=H/2.
      R=200.*RADIUS
CCCC
CCCC  CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
CCCC
      TARC=0.
      DO 110 I=1,1000
      TARC=TARC+H*F(X)
  110 X=X+H
      PRINT 800,L,RADIUS,GFREQ,PH,E1
      PRINT 803,THETA0,PHIP
      PUNCH 808,E1,N,THETA0,PH
```

```
CCCC
CCCC   THROUGH 205 - CALCULATES NUMBER OF ELEMENTS IN EACH SUB ARRAY
CCCC     AND ARC LENGTH BETWEEN THEM
CCCC
       NUM=N
       MM=2
   203 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
      CE))
       X2=RADIUS
   207 NP(MM)=PI*X/LAMBDA
       NP(MM)=2*NP(MM)
       IF(MM-1)240,240,239
   239 IF(NP(MM)-NP(MM-1))240,241,241
   241 NP(MM)=NP(MM-1)-2
   240 NUM=NUM+NP(MM)
       X=NP(MM)*LAMBDA/(2.*PI)
       H=(X2-X)/100.
       Y=X+H/2.
       DO 115 II=1,100
       ARC(MM)=ARC(MM)+H*F(Y)
   115 Y=Y+H
       SUMARC=SUMARC+ARC(MM)
       IF(TARC-SUMARC-LAMBDA/2.)205,205,204
   204.ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
       X1=X-(LAMBDA/2.)*COS(ALPHA)
       X2=X
       X=X1
       MM=MM+1
       GO TO 207
   205 CONTINUE
       MMAX=MM
       MMA=MM+1
CCCC
CCCC   DO 400 LOOP CALCULATES FIELD FOR THETA=M DEGREES
CCCC
```

```
      DO 400 M=1,91
      S7=SIN(THETA0/RAD)
      C7=COS(THETA0/RAD)
      S5=SIN(PHIO)
      C5=COS(PHIO)
      C4=COS(THETA/RAD)
      S4=SIN(THETA/RAD)
      RADIUS
      0.
      A(M)=(0.,0.)
      ALPHA=PI/2.
      THETA1=PI/2.
CCCC
CCCC  DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
CCCC
      DO 250 MM=1,MMAX
      S1=SIN(ALPHA)
      C1=COS(ALPHA)
      S2=SIN(THETA1)
      C2=COS(THETA1)
      N=NP(MM)
CCCC
CCCC  DO 200 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
CCCC
      DO 200 I=1,N
      PHI=2.*PI*(I-1)/N
      S3=SIN(PHI)
      C3=COS(PHI)
      S6=SIN(PHI1-PHI)
      C6=COS(PHI1-PHI)
      S8=SIN(PHI0-PHI)
      C8=COS(PHI0-PHI)
```

```
      PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
      DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
      BETA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)
      IF(BETA)231,208,231
  231 IF(BETA-PI/2.)209,232,232
  232 ELTFAC=0.
      GO TO 211
  208 ELTFAC=1.
      GO TO 211
  209 ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
  211 W=(0.,-1.)*DIST
      Z=(0.,1.)*PSI
      RANGE=K*W
      PHASE=CEXP(Z)
  230 A(M)=A(M)+CEXP(RANGE)*PHASE*ELTFAC
      X=NP(MM+1)*LAMBDA/(2.*PI)
      Z1=SQRT((RADIUS**2.-X**2.)*E)
      ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
  250 THETA1=ATAN2(X,Z1)
      AMAG(M)=CABS(A(M))
      APHASE(M)=ATAN2(AIMAG(A(M)),REAL(A(M)))*RAD
      PRINT 700,THETA,A(M),AMAG(M),APHASE(M)
      PUNCH 807,THETA,A(M),AMAG(M),APHASE(M)
  400 THETA=THETA+0.1
CCCC
CCCC  THROUGH 402 - FINDS MAX FIELD AND ANGLE
CCCC
      D=AMAG(1)
      DO 300 J=2,91
  300 D=AMAX1(D,AMAG(J))
      DO 401 J=1,91
      JJ=J
      IF(AMAG(J)-D)401,402,401
  401 CONTINUE
  402 THETA=FLOAT(JJ-1)
      PRINT 801,D,THETA
```

```
    PRINT 805
    DO 900 I=1,MMAX
900 PRINT 804,I,NP(I)
    PRINT 804,MMA,M1
    PRINT 806,NUM
    GO TO 202
503 STOP
    END
```

```
      PROGRAM THREE(INPUT,OUTPUT,PUNCH)
      COMPLEX RANGE,PHASE,A,W,Z
      REAL LAMBDA,K
       DIMENSION A(91),AMAG(91),APHASE(91),NF(500),ARC(500)
      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
  100 FORMAT(2I5,3F5.0,F5.3,F3.0)
  700 FORMAT(1H ,F5.1,3F15.5,F6.0)
  800 FORMAT(1H1,*SPACING *,I3,*/16 WAVELENGTH*,4X,*RADIUS*,F6.2,* METER
     CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F6.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
     C,F5.3,* OF BASE*)
  801 FORMAT(1H0,*MAXIMUM *,F15.5,* AT *,F3.0,*DEGREES*,/)
  803 FORMAT(1H ,*BEAM COORDINATES--THETA= *,F3.0,2X,*PHI= *,F3.0,/)
  804 FORMAT(1H ,I6,9X,I5)
  805 FORMAT(1H ,*##### NUMBER OF ELEMENTS IN ARRAY #####*,/ ,* CIRCLE N
     CUMBER*,4X,*NUMBER OF ELEMENTS*)
  806 FORMAT(1H0,*TOTAL NUMBER OF ELEMENTS IN ARRAY--*,I8)
  807 FORMAT(F5.1,3F15.5,F6.0)
  808 FORMAT(F5.3,3F15.5,2F3.0)
  999 FORMAT(1H ,*THIS RUN CALCULATES THE VARIATION IN E FOR PHI OF THE
     CFIELD VARYING*)
CCCC
CCCC  DEFINES CONSTANTS TO BE USED IN PROGRAM
CCCC
      FREQ=2.*10.**9.
      PSI=0.
      PI=4.*ATAN(1.0)
      RAD=360./(2.*PI)
      LAMBDA=3.*10.**8./FREQ
      K=2.*PI*FREQ/(3.*10.**8.)
      GFREQ=FREQ/(10.**9.)
CCCC
CCCC  BEGIN CALCULATIONS
CCCC  N=NUMBER OF ELEMENTS AROUND BASE OF ARRAY
CCCC  B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
CCCC  THETA0 AND PHIP ARE BEAM COORDINATES IN DEGREES
```

```
CCCC  E1=RATIO OF HEIGHT OF ARRAY TC RADIUS OF BASE
CCCC  THETA=START ANGLE FOR CALCULATIONS
CCCC
  202 READ 100,N,L,B,THETA0,PHIP,E1,THETA
      IF(N)503,503,502
CCCC
CCCC  THROUGH 702 - INITIALIZES ARRAYS
CCCC
  502 SUMAPC=0.
      DO 701 I=1,500
      APC(I)=0.
  701 NP(I)=0
      DO 702 I=1,91
      A(I)=(0.,0.)
  702 AMAG(I)=APHASE(I)=0.
CCCC
CCCC  CALCULATES REMAINING CCNSTANTS DEPENDENT ON INPUT VARIABLES
CCCC
      E=E1**2
      M1=1
      NP(1)=N
      PHIO=PI*B/N+PI/2.
      PH=PHIO*RAD
      PHI1=PI/2.-PHIP/RAD
      RADIUS=N*LAMBDA*L/(32.*PI)
      H=RADIUS/1000.
      X=H/2.
      R=200.*RADIUS
CCCC
CCCC  CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
CCCC
      TARC=0.
      DO 110 I=1,1000
      TARC=TARC+H*F(X)
  110 X=X+H
      PRINT 800,L,RADIUS,GFREQ,PH,E1
```

```
      PPINT 803,THETA0,PHIP
      PRINT 999
      PUNCH 808,E1,N,THETA0,PH
CCCC
CCCC  THROUGH 205 - CALCULATES NUMBER OF ELEMENTS IN EACH SUB ARRAY
CCCC    AND ARC LENGTH BETWEEN THEM
CCCC
      NUM=N
      MM=2
  203 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
     CE))
      X2=RADIUS
  207 NP(MM)=PI*X/LAMBDA
      NP(MM)=2*NP(MM)
      IF(MM-1)240,240,239
  239 IF(NP(MM)-NP(MM-1))240,241,241
  241 NP(MM)=NP(MM-1)+2
  240 NUM=NUM+NP(MM)
      X=NP(MM)*LAMBDA/(2.*PI)
      H=(X2-X)/100.
      Y=X+H/2.
      DO 115 II=1,100
      ARC(MM)=ARC(MM)+H*F(Y)
  115 Y=Y+H
      SUMARC=SUMARC+ARC(MM)
      IF(TARC-SUMARC-LAMBDA/2.)205,205,204
  204 ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
      X1=X-(LAMBDA/2.)*COS(ALPHA)
      X2=X
      X=X1
      MM=MM+1
      GO TO 207
  205 CONTINUE
      MMAX=MM
      MMA=MM+1
```

```
CCCC
CCCC  DO 400 LOOP CALCULATES FIELD FOR THETA=M DEGREES
CCCC
      DO 400 M=1,90
      THETA0=THETA
      S7=SIN(THETA0/RAD)
      C7=COS(THETA0/RAD)
      S5=SIN(PHI0)
      C5=COS(PHI0)
      C4=COS(THETA/RAD)
      S4=SIN(THETA/RAD)
      X=RADIUS
      Z1=0.
      A(M)=(0.,0.)
      ALPHA=PI/2.
      THETA1=PI/2.
CCCC
CCCC  DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
CCCC
      DO 250 MM=1,MMAX
      S1=SIN(ALPHA)
      C1=COS(ALPHA)
      S2=SIN(THETA1)
      C2=COS(THETA1)
      N=NP(MM)
CCCC
CCCC  DO 200 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
CCCC
      DO 200 I=1,N
      PHI=2.*PI*(I-1)/N
      S3=SIN(PHI)
      C3=COS(PHI)
      S6=SIN(PHI1-PHI)
      C6=COS(PHI1-PHI)
      S8=SIN(PHI0-PHI)
      C8=COS(PHI0-PHI)
```

```
      PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
      DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
      BETA=ACOS((S1*(R*S4*C5-X)+C1*(R*C4-Z1))/DIST)
      IF(BETA)231,208,231
  231 IF(BETA-PI/2.)209,232,232
  232 ELTFAC=0.
      GO TO 211
  208 ELTFAC=1.
      GO TO 211
  209 ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
  211 W=(0.,-1.)*DIST
      Z=(0.,1.)*PSI
      RANGE=K*W
      PHASE=CEXP(Z)
  200 A(M)=A(M)+CEXP(RANGE)*PHASE*ELTFAC
      X=NP(MM+1)*LAMBDA/(2.*PI)
      Z1=SQRT((RADIUS**2.-X**2.)*E)
      ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
  250 THETA1=ATAN2(X,Z1)
      AMAG(M)=CABS(A(M))
      APHASE(M)=ATAN2(AIMAG(A(M)),REAL(A(M)))*RAD
      PH=PHIO*RAD
      PRINT 700,PH,A(M),AMAG(M),APHASE(M)
      PUNCH 807,PH,A(M),AMAG(M),APHASE(M)
  400 PHIO=PHIO+1.0/RAD
CCCC
CCCC  THROUGH 402 - FINDS MAX FIELD AND ANGLE
CCCC
      D=AMAG(1)
      DO 300 J=2,91
  300 D=AMAX1(D,AMAG(J))
      DO 401 J=1,91
      JJ=J
      IF(AMAG(J)-D)401,402,401
  401 CONTINUE
```

```
432 THETA=FLOAT(JJ-1)
    PRINT 801,D,THETA
    PRINT 805
    DO 900 I=1,MMAX
900 PRINT 804,I,NP(I)
    PRINT 804,MMA,M1
    PRINT 806,NUM
    GO TO 202
503 STOP
    END
```

```
      PROGRAM FOUR(INPUT,OUTPUT,PUNCH)
      COMPLEX RANGE,PHASE,A,W,Z
      REAL LAMBDA,K
       DIMENSION A(91),AMAG(91),APHASE(91),NF(500),ARC(500)
      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
  100 FORMAT(2I5,3F5.0,F5.3,F3.0)
  700 FORMAT(1H ,F6.1,3F15.5,F6.0)
  800 FORMAT(1H1,*SPACING *,I3,*/16 WAVELENGTH*,4X,*RADIUS*,F6.2,* METER
     CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F6.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
     C,F5.3,* OF BASE*)
  801 FORMAT(1H0,*MAXIMUM *,F15.5,* AT *,F3.0,*DEGREES*,/)
  803 FORMAT(1H ,*BEAM COORDINATES--THETA= *,F3.0,2X,*PHI= *,F3.0,/)
  804 FORMAT(1H ,I6,9X,I5)
  805 FORMAT(1H ,*##### NUMBER OF ELEMENTS IN ARRAY #####*,/ ,* CIRCLE N
     CUMBER*,4X,*NUMBER OF ELEMENTS*)
  806 FORMAT(1H0,*TOTAL NUMBER OF ELEMENTS IN ARRAY--*,I8)
  807 FORMAT(F6.1,3F15.5,F6.0)
  808 FORMAT(F5.3,3F15.5,2F3.0)
CCCC
CCCC  DEFINES CONSTANTS TO BE USED IN PROGRAM
CCCC
      FREQ=2.*10.**9.
      PSI=0.
      PI=4.*ATAN(1.0)
      RAD=360./(2.*PI)
      LAMBDA=3.*10.**8./FREQ
      K=2.*PI*FREQ/(3.*10.**8.)
      GFREQ=FREQ/(10.**9.)
CCCC
CCCC  BEGIN CALCULATIONS
CCCC  N=NUMBER OF ELEMENTS AROUND BASE OF ARRAY
CCCC  B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
CCCC  THETA0 AND PHIP ARE BEAM COORDINATES IN DEGREES
CCCC  E1=RATIO OF HEIGHT OF ARRAY TO RADIUS OF BASE
CCCC  THETA=START ANGLE FOR CALCULATIONS
CCCC
```

```
  202 READ 100,N,L,B,THETA0,PHIP,E1,THETA
      IF(N)503,503,502
CCCC
CCCC  THROUGH 702 - INITIALIZES ARRAYS
CCCC
  502 SUMARC=0.
      DO 701 I=1,500
      ARC(I)=0.
  701 NP(I)=0
      DO 702 I=1,91
      A(I)=(0.,0.)
  702 AMAG(I)=APHASE(I)=0.
CCCC
CCCC  CALCULATES REMAINING CONSTANTS DEPENDENT ON INPUT VARIABLES
CCCC
      E=E1**2
      M1=1
      NP(1)=N
      PHIO=PI*B/N+PI/2.
      PH=PHIO*RAD
      PHI1=PI/2.-PHIP/RAD
      RADIUS=N*LAMBDA*L/(32.*PI)
      H=RADIUS/1000.
      X=H/2.
      R=200.*RADIUS
CCCC
CCCC  CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
CCCC
      TARC=0.
      DO 110 I=1,1000
      TARC=TARC+H*F(X)
  110 X=X+H
      PRINT 800,L,RADIUS,GFREQ,PH,E1
      PRINT 803,THETA0,PHIP
      PUNCH 808,E1,N,THETA0,PH
```

```
CCCC
CCCC  THROUGH 205 - CALCULATES NUMBER OF ELEMENTS IN EACH SUB ARRAY
CCCC    AND ARC LENGTH BETWEEN THEM
CCCC
      NUM=N
      MM=2
  203 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
     CE))
      X2=RADIUS
  207 NP(MM)=PI*X/LAMBDA
      NP(MM)=2*NP(MM)
      IF(MM-1)240,240,239
  239 IF(NP(MM)-NP(MM-1))240,241,241
  241 NP(MM)=NP(MM-1)-2
  240 NUM=NUM+NP(MM)
      X=NP(MM)*LAMBDA/(2.*PI)
      H=(X2-X)/100.
      Y=X+H/2.
      DO 115 II=1,100
      ARC(MM)=ARC(MM)+H*F(Y)
  115 Y=Y+H
      SUMARC=SUMARC+ARC(MM)
      IF(TARC-SUMARC-LAMBDA/2.)205,205,204
  204 ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
      X1=X-(LAMBDA/2.)*COS(ALPHA)
      X2=X
      X=X1
      MM=MM+1
      GO TO 207
  205 CONTINUE
      MMAX=MM
      MMA=MM+1
CCCC
CCCC  DO 400 LOOP CALCULATES FIELD FOR THETA=M DEGREES
CCCC
      DO 400 M=1,91
```

```
      THETA0=THETA
      S7=SIN(THETA0/RAD)
      C7=COS(THETA0/RAD)
      S5=SIN(PHIO)
      C5=COS(PHIO)
      C4=COS(THETA/RAD)
      S4=SIN(THETA/RAD)
      X=RADIUS
      Z1=0.
      A(M)=(0.,0.)
      ALPHA=PI/2.
      THETA1=PI/2.
CCCC
CCCC  DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
CCCC
      DO 250 MM=1,MMAX
      S1=SIN(ALPHA)
      C1=COS(ALPHA)
      S2=SIN(THETA1)
      C2=COS(THETA1)
      N=NP(MM)
CCCC
CCCC  DO 200 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
CCCC
      DO 200 I=1,N
      PHI=2.*PI*(I-1)/N
      S3=SIN(PHI)
      C3=COS(PHI)
      S6=SIN(PHI1-PHI)
      C6=COS(PHI1-PHI)
      S8=SIN(PHIO-PHI)
      C8=COS(PHIO-PHI)
      PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
      DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
      BETA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)
      IF(BETA)231,208,231
```

```
231 IF(BETA-PI/2.)209,232,232
232 ELTFAC=0.
    GO TO 211
208 ELTFAC=1.
    GO TO 211
209 ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
211 W=(0.,-1.)*DIST
    Z=(0.,1.)*PSI
    RANGE=K*W
    PHASE=CEXP(Z)
200 A(M)=A(M)+CEXP(RANGE)*PHASE*ELTFAC
    X=NP(MM+1)*LAMBDA/(2.*PI)
    Z1=SQRT((RADIUS**2.-X**2.)*E)
    ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
250 THETA1=ATAN2(X,Z1)
    AMAG(M)=CABS(A(M))
    APHASE(M)=ATAN2(AIMAG(A(M)),REAL(A(M)))*RAD
    PH=PHIO*RAD
    PRINT 799,PH,A(M),AMAG(M),APHASE(M)
    PUNCH 807,PH,A(M),AMAG(M),APHASE(M)
400 PHIO=PHIO+0.1/RAD
CCCC
CCCC THROUGH 402 - FINDS MAX FIELD AND ANGLE
CCCC
    D=AMAG(1)
    DO 300 J=2,91
300 D=AMAX1(D,AMAG(J))
    DO 401 J=1,91
    JJ=J
    IF(AMAG(J)-D)401,402,401
401 CONTINUE
402 THETA=FLOAT(JJ-1)
    PRINT 801,D,THETA
    PRINT 805
    DO 900 I=1,MMAX
900 PRINT 804,I,NP(I)
```

```
      PRINT 804,MMA,M1
      PRINT 806,NUM
      GO TO 202
  503 STOP
      END
```

```
      PROGRAM BEAM(INPUT,OUTPUT,PUNCH)
      COMPLEX RANGE,PHASE,A,W,Z
      REAL LAMBDA,K
       DIMENSION A(91),AMAG(91),APHASE(91),NP(500),ARC(500)
      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
  100 FORMAT(2I5,3F5.0,F5.3,F3.0)
  700 FORMAT(1H ,F5.1,3F15.5,F6.0)
  800 FORMAT(1H1,*SPACING *,I3,*/16 WAVELENGTH*,4X,*RADIUS*,F6.2,* METER
     CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F5.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
     C,F5.3,* OF BASE*)
  801 FORMAT(1H-,*MAXIMUM *,F15.5,* AT *,F3.0,*DEGREES*,/)
  803 FORMAT(1H ,*BEAM COORDINATES--THETA= *,F3.0,2X,*PHI= *,F3.0,/)
  804 FORMAT(1H ,I6,9X,I5)
  805 FORMAT(1H ,*##### NUMBER OF ELEMENTS IN ARRAY #####*,/ ,* CIRCLE N
     CUMBER*,4X,*NUMBER OF ELEMENTS*)
  806 FORMAT(1H0,*TOTAL NUMBER OF ELEMENTS IN ARRAY--*,I8)
  807 FORMAT(F6.1,3F15.5,F6.0)
  808 FORMAT(F5.3,3F15.5,2F3.0)
CCCC
CCCC  DEFINES CONSTANTS TO BE USED IN PROGRAM
CCCC
      FREQ=2.*10.**9.
      PSI=0.
      PI=4.*ATAN(1.0)
      RAD=360./(2.*PI)
      LAMBDA=3.*10.**8./FREQ
      K=2.*PI*FREQ/(3.*10.**8.)
      GFREQ=FREQ/(10.**9.)
CCCC
CCCC  BEGIN CALCULATIONS
CCCC  N=NUMBER OF ELEMENTS AROUND BASE OF ARRAY
CCCC  B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
CCCC  THETA0 AND PHIP ARE BEAM COORDINATES IN DEGREES
CCCC  E1=RATIO OF HEIGHT OF ARRAY TO RADIUS OF BASE
CCCC  THETA=START ANGLE FOR CALCULATIONS
```

```
CCCC
  202 READ 100,N,L,B,THETA0,PHIP,E1,THETA
      IF(N)503,503,502
CCCC
CCCC  THROUGH 702 - INITIALIZES ARRAYS
CCCC
  502 SUMARC=0.
      DO 701 I=1,500
      ARC(I)=0.
  701 NP(I)=0
      DO 702 I=1,91
      A(I)=(0.,0.)
  702 AMAG(I)=APHASE(I)=0.
CCCC
CCCC  CALCULATES REMAINING CCNSTANTS DEPENDENT ON INPUT VARIABLES
CCCC
      E=E1**2
      M1=1
      NP(1)=N
      PHIO=PI*B/N+PI/2.
      PH=PHIO*RAD
      PHI1=PI/2.-PHIP/RAD
      RADIUS=N*LAMBDA*L/(32.*PI)
      H=RADIUS/1000.
      X=H/2.
      R=200.*RADIUS
CCCC
CCCC  CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
CCCC
      TARC=0.
      DO 110 I=1,1000
      TARC=TARC+H*F(X)
  110 X=X+H
      PRINT 800,L,RADIUS,GFREQ,PH,E1
      PRINT 803,THETA0,PHIP
      PUNCH 808,E1,N,THETA0,PH
CCCC
```

```
CCCC  THROUGH 205 - CALCULATES NUMBER OF ELEMENTS IN EACH SUB ARRAY
CCCC    AND ARC LENGTH BETWEEN THEM
CCCC
      NUM=N
      MM=2
  203 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
     CE))
      X2=RADIUS
  207 NP(MM)=PI*X/LAMBDA
      NP(MM)=2*NP(MM)
      IF(MM-1)240,240,239
  239 IF(NP(MM)-NP(MM-1))240,241,241
  241 NP(MM)=NP(MM-1)-2
  240 NUM=NUM+NP(MM)
      X=NP(MM)*LAMBDA/(2.*PI)
      H=(X2-X)/100.
      Y=X+H/2.
      DO 115 II=1,100
      ARC(MM)=ARC(MM)+H*F(Y)
  115 Y=Y+H
      SUMARC=SUMARC+ARC(MM)
      IF(TARC-SUMARC-LAMBDA/2.)205,205,204
  204 ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
      X1=X-(LAMBDA/2.)*COS(ALPHA)
      X2=X
      X=X1
      MM=MM+1
      GO TO 207
  205 CONTINUE
      MMAX=MM
      MMA=MM+1
CCCC
CCCC  DO 400 LOOP CALCULATES FIELD FOR THETA=M DEGREES
CCCC
      DO 400 M=1,91
      THETA0=THETA
```

```
      S7=SIN(THETA0/RAD)
      C7=COS(THETA0/RAD)
      S5=SIN(PHIO)
      C5=COS(PHIO)
      C4=COS(THETA/RAD)
      S4=SIN(THETA/RAD)
      X=RADIUS
      Z1=0.
      A(MM)=(0.,0.)
      ALPHA=PI/2.
      THETA1=PI/2.
CCCC
CCCC  DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
CCCC
      DO 250 MM=1,MMAX
      S1=SIN(ALPHA)
      C1=COS(ALPHA)
      S2=SIN(THETA1)
      C2=COS(THETA1)
      N=NP(MM)
CCCC
CCCC  DO 200 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
CCCC
      DO 200 I=1,N
      PHI=2.*PI*(I-1)/N
      S3=SIN(PHI)
      C3=COS(PHI)
      S6=SIN(PHI1-PHI)
      C6=COS(PHI1-PHI)
      S8=SIN(PHIO-PHI)
      C8=COS(PHIO-PHI)
      PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
      DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
      BETA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)
      IF(BETA)231,208,231
```

```
 231 IF(BETA-PI/2.)209,232,232
 232 ELTFAC=0.
     GO TO 211
 208 ELTFAC=1.
     GO TO 211
 209 ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
 211 W=(0.,-1.)*DIST
     Z=(0.,1.)*PSI
     RANGE=K*W
     PHASE=CEXP(Z)
 200 A(M)=A(M)+CEXP(RANGE)*PHASE*ELTFAC
     X=NP(MM+1)*LAMBDA/(2.*PI)
     Z1=SQRT((RADIUS**2.-X**2.)*E)
     ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
 250 THETA1=ATAN2(X,Z1)
     AMAG(M)=CABS(A(M))
     APHASE(M)=ATAN2(AIMAG(A(M)),REAL(A(M)))*RAD
     PRINT 700,THETA,A(M),AMAG(M),APHASE(M)
     PUNCH 807,THETA,A(M),AMAG(M),APHASE(M)
 400 THETA=THETA+1.
CCCC
CCCC THROUGH 402 - FINDS MAX FIELD AND ANGLE
CCCC
     D=AMAG(1)
     DO 300 J=2,91
 300 D=AMAX1(D,AMAG(J))
     DO 401 J=1,91
     JJ=J
     IF(AMAG(J)-D)401,402,401
 401 CONTINUE
 402 THETA=FLOAT(JJ-1)
     PRINT 801,D,THETA
     PRINT 805
     DO 900 I=1,MMAX
 900 PRINT 804,I,NP(I)
     PRINT 804,MMA,M1
```

```
      PRINT 806,NUM
      GO TO 202
  503 STOP
      END
```

```
      PROGRAM OPTIMUM(INPUT,OUTPUT)
      COMPLEX A,W,Z,RANGE,PHASE
      REAL LAMBDA,K,L
      DIMENSION A(2),AMAG(2),NP(500),ARC(500)
      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
CCCC
CCCC  DEFINE CONSTANTS TO BE USED IN PROGRAM
CCCC
      I1=I2=I3=I4=I5=0
      FREQ=2.*10.**9.
      PI=4.*ATAN(1.0)
      RAD=360./(2.*PI)
      LAMBDA=3.*10.**8./FREQ
      K=2.*PI*FREQ/(3.*10.**8.)
      E1=1.1
      RATIO=1.
CCCC
CCCC  INITIALIZE VARIABLES
CCCC
  190 SUMARC=THETA=THETAO=PSI=0.
      N=128
      DO 100 I=1,500
      ARC(I)=0.
  100 NP(I)=0
      DO 110 I=1,2
      A(I)=(0.,0.)
  110 AMAG(I)=0.
      M1=1
      NP(1)=N
      PHIO=PHI1=PI/2.
      RADIUS=N*LAMBDA/(2.*PI)
      R=200.*RADIUS
CCCC
-CCCC  CALCULATE GEOMETRY OF ARRAY
CCCC
      E=E1**2
```

```
      H=RADIUS/1000.
      X=H/2.
CCCC
CCCC  CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
CCCC
  200 TARC=0.
      DO 201 I=1,1000
      TARC=TARC+H*F(X)
  201 X=X+H
      NUM=N
      MM=2
  202 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
     CE))
      X2=RADIUS
  209 NP(MM)=X*K/2.
      NP(MM)=2*NP(MM)
      IF(MM-1)204,204,203
  203 IF(NP(MM)-NP(MM-1))204,205,205
  205 NP(MM)=NP(MM-1)-2
  204 NUM=NUM+NP(MM)
      X=NP(MM)/K
      H=(X2-X)/100.
      Y=X+H/2.
      DO 206 II=1,100
      ARC(MM)=ARC(MM)+H*F(Y)
  206 Y=Y+H
      SUMARC=SUMARC+ARC(MM)
      IF(TARC-SUMARC-LAMBDA/2.)207,207,208
  208 ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
      X1=X-(LAMBDA/2.)*COS(ALPHA)
      X2=X
      X=X1
      MM=MM+1
      GO TO 209
  207 CONTINUE
      MMAX=MM
```

```
CCCC
CCCC  CALCULATE FIELD AT THETA = 0,90 DEGREES
CCCC
      S5=1.
      C5=0.
      DO 290 M=1,2
      IF(M.EQ.1) THETA=0.
      IF(M.EQ.2) THETA=PI/2.
      S7=S4=SIN(THETA)
      C7=C4=COS(THETA)
      X=RADIUS
      Z1=0.
      A(M)=(0.,0.)
      ALPHA=PI/2.
      THETA1=PI/2.
CCCC
CCCC  DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUBARRAY
CCCC
      DO 250 MM=1,MMAX
      S1=SIN(ALPHA)
      C1=COS(ALPHA)
      S2=SIN(THETA1)
      C2=COS(THETA1)
      N=NP(MM)
CCCC
CCCC  DO 240 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
CCCC
      DO 240 I=1,N
      PHI=2.*PI*(I-1)/N
      S3=SIN(PHI)
      C3=COS(PHI)
      S6=SIN(PHI1-PHI)
      C6=COS(PHI1-PHI)
      S8=SIN(PHIC-PHI)
      C8=COS(PHIC-PHI)
```

```
      PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
      DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
      BETA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)

      IF(BETA)231,233,231
  231 IF(BETA-PI/2.)234,232,232
  232 ELTFAC=0.
      GO TO 211
  233 ELTFAC=1.
      GO TO 211
  234 ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
  211 W=(0.,-1.)*DIST
      Z=(0.,1.)*PSI
      RANGE=K*W
      PHASE=CEXP(Z)
  240 A(M)=A(M)+CEXP(RANGE)*PHASE*ELTFAC
      X=NP(MM+1)/K
      Z1=SQRT((RADIUS**2.-X**2.)*E)
      ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
  250 THETA1=ATAN2(X,Z1)
      AMAG(M)=CABS(A(M))
  290 CONTINUE
CCCC
CCCC  TEST FOR OPTIMUM VALUE OF E1. IF NOT OPTIMUM STEP E1 AND GO TO 190
CCCC
      AM=AMAG(2)/AMAG(1)
      IF(RATIO-AM)301,302,302
  301 I2=1
      IF(I1)304,304,305
  304 E1=E1+0.1
      RATIO=AMAG(2)/AMAG(1)
      GO TO 190
  305 IF(I4)306,306,307
  306 E1=E1-0.01
      I3=1
      RATIO=AMAG(2)/AMAG(1)
```

```
      GO TO 190
  307 E1=E1+0.001
      I5=1
      RATIO=AMAG(2)/AMAG(1)
      GO TO 190
  302 IF(I2)308,308,309
  308 E1=E1+0.1
      RATIO=AMAG(2)/AMAG(1)
      GO TO 190
  309 IF(I3)310,310,311
  310 E1=E1-0.01
      I1=1
      RATIO=AMAG(2)/AMAG(1)
      GO TO 190
  311 IF(I5)312,312,313
  312 I4=1
      E1=E1+0.001
      RATIO=AMAG(2)/AMAG(1)
      GO TO 190
  313 CONTINUE
CCCC
CCCC  PRINT GEOMETRY OF ARRAY
CCCC
      HEIGHT=E1*RADIUS
      PRINT 700,E1,HEIGHT,RADIUS,NUM
  700 FORMAT(1H ,*OPTIMUM VALUE OF E1 -- *,F5.3/* HEIGHT OF ARRAY -- *,F
     C8.4,* METERS*/* RADIUS OF BASE OF ARRAY -- *,F6.4,* METERS*/* TOTA
     CL NUMBER OF ELEMENTS IN ARRAY -- *,I6)
      PRINT 805
  805 FORMAT(1H ,*##### NUMBER OF ELEMENTS IN ARRAY #####*,/ ,* CIRCLE N
     CUMBER*,4X,*NUMBER OF ELEMENTS*)
      DO 900 I=1,MMAX
  900 PRINT 701,I,NP(I)
  701 FORMAT(1H ,I5,9X,I5)
      END
```

Appendix C

Geometry of Array

Circle Number	Number of Elements
1	128
2	126
3	124
4	122
5	120
6	118
7	116
8	114
9	112
10	110
11	108
12	106
13	104
14	102
15	100
16	98
17	96
18	94
19	92
20	90
21	88
22	86
23	84
24	82
25	80

Circle Number	Number of Elements
26	78
27	76
28	74
29	72
30	70
31	68
32	66
33	64
34	62
35	60
36	58
37	56
38	54
39	50
40	46
41	42
42	38
43	34
44	30
45	26
46	22
47	18
48	14
49	10
50	6
51	2

Height of Array --- 7.6 meters

Radius of Base of Array --- 3.06 meters

Total Number of Elements in Array --- 3796

Vita

Thomas Burke Markham was born on 1 March 1941 in Memphis, Tennessee. He graduated from Clarksville High School in Clarksville, Tennessee, in 1958. He attended Austin Peay State University for two years prior to enlisting in the United States Air Force in 1960. After completing technical school he was assigned to Vandenberg Air Force Base, California, from 1961 to 1965 as a Missile Instrumentation and Range Safety technician with the Strategic Air Command's 4392nd Communications Squadron. In 1965 he entered Arizona State University under the Airmen's Education and Commissioning Program from which he received a Bachelor of Science in Engineering degree. Upon completion of Officer Training School he was commissioned a Second Lieutenant in the Air Force Reserve. He served as a Satellite Operations Control Officer from 1967 to 1970. He attended the Air Force Institute of Technology where he received the degree of Master of Science in Electrical Engineering whereupon he was assigned to Vandenberg Air Force Base as a Satellite Vehicle Director.

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